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Acceleration / Deceleration Factor

Ratio between the forces required to move an object and the weight of the object, given as a percentage of gravity.

Accel/Decelerating

1. Friction coefficient of an accel/decelerating object, decimal.

\[ \mu = \frac{a}{g} \]

\( a = \text{Accel / Decel rate, ft/sec}^2 \)
\( g = \text{Gravitational constant, 32.2 ft/sec}^2 \)

Percentage of Acceptability

2. Determination of an acceptable friction coefficient range, percent. The percentage acceptability should be within 5%.

\[ P_a = \frac{(L\mu - S\mu)100}{S\mu} \]

\( L\mu = \text{Largest friction coefficient during testing, decimal} \)
\( S\mu = \text{Smallest friction coefficient during testing, decimal} \)

Equivalent

3. Equivalent friction coefficient of a level surface from a grade, decimal.

\[ \mu_e = \mu - \sin(Tan^{-1}(m)) / \cos(Tan^{-1}(m)) \]

\( \mu = \text{Friction coefficient of grade, decimal} \)
\( m = \text{Grade, decimal (negative value (-) for decline)} \)

4. Equivalent friction coefficient of a grade from a level surface, decimal.

\[ \mu_e = \sin(Tan^{-1}(m)) + \mu \times \cos(Tan^{-1}(m)) \]

\( \mu = \text{Level friction coefficient, decimal} \)
\( m = \text{Grade, decimal (negative value (-) for decline)} \)
5. Equivalent friction coefficient of a grade from a level surface, decimal.

\[ \mu_e = \frac{\mu \pm m}{\sqrt{1 \pm m^2}} \]

\( \mu = \text{Level friction coefficient, decimal} \)
\( m = \text{Grade, decimal} \) \{(-) for decline, (+) for incline\}

6. Equivalent deceleration factor for a straight line skid on several surfaces, decimal.

\[ f_e = \frac{d_1 f_1 + d_2 f_2 + d_3 f_3 + d_4 f_4}{d_1 + d_2 + d_3 + d_4} \]

\( d_1 \rightarrow d_4 = \text{Distance of each individual surface, ft} \)
\( f_1 \rightarrow f_4 = \text{Deceleration factor of each individual surface, decimal} \)

7. Equivalent deceleration factor for a two axle vehicle during a straight line skid, knowing the center of mass location \((x_{Fi} \text{ and } z_i)\). Center of mass utilized as a decimal fraction of the wheelbase, decimal.

\[ f_e = \frac{f_F - x_{Fi}(f_F - f_R)}{1 - z_i(f_F - f_R)} \]

\( f_F = \text{Front deceleration factor, decimal} \)
\( f_R = \text{Rear deceleration factor, decimal} \)
\( x_{Fi} = \text{Longitudinal center of mass from the front axle, decimal} \)
\( z_i = \text{Vertical center of mass height, decimal} \)

8. Adjusted deceleration factor for braking with a grade less than 11.9%, decimal.

\[ f = \mu n \pm m \]

\( \mu = \text{Level friction coefficient, decimal} \)
\( n = \text{Braking efficiency, decimal} \)
\( m = \text{Grade, decimal} \) \{(-) for decline, (+) for incline\}
Stop, From or To

9. Accel/deceleration factor from or to stop, decimal.

\[ f = S^2 / (30dn) \]

\[ \begin{align*}
S &= \text{Speed, mi/hr} \\
d &= \text{Distance, ft} \\
n &= \text{Braking efficiency, decimal (deceleration only)}
\end{align*} \]

10. Accel/deceleration factor from or to a stop over a unit of time, decimal.

\[ f = 1.466S / (gT) \]

\[ \begin{align*}
S &= \text{Speed, mi/hr} \\
T &= \text{Time, sec} \\
g &= \text{Gravitational constant, 32.2 ft/sec}^2
\end{align*} \]

11. Accel/deceleration factor from or to a stop, decimal.

\[ f = V^2 / (2gdn) \]

\[ \begin{align*}
V &= \text{Velocity, ft/sec} \\
d &= \text{Distance, ft} \\
g &= \text{Gravitational constant, 32.2 ft/sec}^2 \\
n &= \text{Braking efficiency, decimal (deceleration only)}
\end{align*} \]

Stop, From or To; Time

12. Accel/deceleration factor from or to a stop over a unit of time, decimal.

\[ f = V / (gT) \]

\[ \begin{align*}
V &= \text{Velocity, ft/sec} \\
T &= \text{Time, sec} \\
g &= \text{Gravitational constant, 32.2 ft/sec}^2
\end{align*} \]

13. Accel/deceleration factor from or to a stop over a unit of time, decimal.

\[ f = 2d / (gT^2) \]

\[ \begin{align*}
d &= \text{Distance, ft} \\
T &= \text{Time, sec} \\
g &= \text{Gravitational constant, 32.2 ft/sec}^2
\end{align*} \]
14. Accel/deceleration factor from or to a stop over a unit of time, \textit{decimal}.

\[ f = \frac{d}{16.1T^2} \]

\( d = \) Distance, ft
\( T = \) Time, sec

\textbf{Acceleration}

15. Acceleration factor from one speed to another over a determined distance, \textit{decimal}.

\[ f = \frac{S_f^2 - S_o^2}{30d} \]

\( S_f = \) Speed final, mi/hr
\( S_o = \) Speed initial, mi/hr
\( d = \) Distance, ft

16. Acceleration factor from one velocity to another over a determined distance, \textit{decimal}.

\[ f = \frac{V_f^2 - V_o^2}{2gd} \]

\( V_f = \) Velocity final, ft/sec
\( V_o = \) Velocity initial, ft/sec
\( d = \) Distance, ft
\( g = \) Gravitational constant, 32.2 ft/sec²

17. Acceleration factor from one velocity to another over a unit of time, \textit{decimal}.

\[ f = \frac{V_f - V_o}{gT} \]

\( V_f = \) Velocity final, ft/sec
\( V_o = \) Velocity initial, ft/sec
\( T = \) Time, sec
\( g = \) Gravitational constant, 32.2 ft/sec²

18. Acceleration factor over a determined distance and a unit of time, \textit{decimal}.

\[ f = \frac{d - V_o T}{T^2 g / 2} \]

\( V_o = \) Velocity initial, ft/sec
\( d = \) Distance, ft
\( T = \) Time, sec
\( g = \) Gravitational constant, 32.2 ft/sec²
19. Acceleration factor over a determined distance and a unit of time, **decimal**.

\[ f = \frac{d}{(T^2 g / 2)} - \frac{Vo}{(Tg / 2)} \]

- \( Vo \) = Velocity initial, ft/sec
- \( d \) = Distance, ft
- \( T \) = Time, sec
- \( g \) = Gravitational constant, 32.2 ft/sec\(^2\)

**Deceleration**

20. Friction coefficient of a decelerating object knowing the weight and force applied, **decimal**.

\[ \mu = \frac{F}{W} \]

- \( F \) = Force, lb
- \( W \) = Total static weight, lb

21. Deceleration factor from one speed to another over a determined distance, **decimal**.

\[ f = \frac{So^2 - Sf^2}{30d} \]

- \( So \) = Speed initial, mi/hr
- \( Sf \) = Speed final, mi/hr
- \( d \) = Distance, ft

22. Deceleration factor from one velocity to another over a determined distance, **decimal**.

\[ f = \frac{Vo^2 - Vf^2}{2gd} \]

- \( Vo \) = Velocity initial, ft/sec
- \( Vf \) = Velocity final, ft/sec
- \( d \) = Distance, ft
- \( g \) = Gravitational constant, 32.2 ft/sec\(^2\)

23. Deceleration factor from one velocity to another over a unit of time, **decimal**.

\[ f = \frac{Vo - Vf}{gT} \]

- \( Vo \) = Velocity initial, ft/sec
- \( Vf \) = Velocity final, ft/sec
- \( T \) = Time, sec
- \( g \) = Gravitational constant, 32.2 ft/sec\(^2\)
24. Deceleration factor from one speed to another over a unit of time, \textit{decimal}.

\[ f = (S_0 - S_f) / (21.96T) \]

\( S_0 = \text{Speed initial, mi/hr} \)

\( S_f = \text{Speed final, mi/hr} \)

\( T = \text{Time, sec} \)

25. Deceleration factor over a determined distance and a unit of time, \textit{decimal}.

\[ f = V_0 / (Tg / 2) - d / (T^2g / 2) \]

\( V_0 = \text{Velocity initial, ft/sec} \)

\( d = \text{Distance, ft} \)

\( T = \text{Time, sec} \)

\( g = \text{Gravitational constant, 32.2 ft/sec}^2 \)

\textbf{Lateral}

26. Lateral acceleration factor needed to maintain the radius of a level curve at a determined speed, \textit{decimal}.

\[ f_y = S^2 / (14.97r) \]

\( S = \text{Speed, mi/hr} \)

\( r = \text{Radius of roadway, ft} \)

27. Lateral acceleration factor of a vehicle negotiating a level curve at a determined speed with an unknown radius at the center of mass, \textit{decimal}.

\[ f_y = \frac{S^2}{14.97(r - 0.5tw)} \]

\( S = \text{Speed, mi/hr} \)

\( r = \text{Radius of yaw mark, ft} \)

\( tw = \text{Track width, ft} \)
28. Lateral acceleration factor of a vehicle negotiating a banked curve at a determined speed with a known radius at the center of mass, **decimal**.

\[ f_y = \left(\frac{(S/3.86)^2}{r-e}\right) / \left(1 + \left(\frac{S/3.86}{e/r}\right)^2\right) \]

- **S** = Speed, mi/hr
- **r** = Radius traveled by center of mass, ft
- **e** = Superelevation of curve, decimal (negative value (-) for decline)

29. Lateral acceleration factor needed to maintain the radius of a level curve at a determined velocity, **decimal**.

\[ f_y = \frac{V^2}{rg} \]

- **V** = Velocity, ft/sec
- **r** = Radius of roadway, ft
- **g** = Gravitational constant, 32.2 ft/sec²

30. Lateral acceleration factor of a vehicle negotiating a level curve at a determined velocity with an unknown radius at the center of mass, **decimal**.

\[ f_y = \frac{V^2}{g(r-0.5tw)} \]

- **V** = Velocity, ft/sec
- **r** = Radius of yaw mark, ft
- **tw** = Track width, ft
- **g** = Gravitational constant, 32.2 ft/sec²
31. Lateral equivalent deceleration factor for a vehicle sliding in a yaw on different friction surfaces, decimal.

\[ f_{ey} = \frac{(f_i + f_o)tw}{z(f_i + f_o) + tw} \]

\( tw = \) Track width, ft

\( f_i = \) Braking coefficient for the surface on which the inner wheels are rolling, decimal

\( f_o = \) Braking coefficient for the surface on which the outer wheels are rolling, decimal

\( z = \) Vertical center of mass height, ft

Weinberg

Lateral Stability

32. Determine a vehicle's lateral stability. The friction coefficient of the roadway must be greater than the value of the solution for the vehicle to rollover, decimal.

\[ f_y = \frac{tw}{2z} \]

\( tw = \) Track width, in

\( z = \) Vertical center of mass height, in

Rolling Resistance

33. Rolling resistance coefficient for bias or radial tires, decimal.

\[ f_{roll} = a + \frac{0.15}{p} + \frac{b}{p} \left( \frac{S}{100} \right)^2 \]

\( S = \) Speed, mi/hr

\( p = \) Tire inflation pressure, lb/in²

Limpert

Radial:

\( a = 0.005 \)

\( b = 0.67 \)

Bias Ply:

\( a = 0.009 \)

\( b = 1.0 \)
34. Rolling resistance coefficient for radial tires on heavy trucks, decimal.

\[ f_{roll} = (0.0041 + 0.000041V)f \]

\( V = \) Velocity, ft/sec
\( f = \) Friction coefficient, decimal

\( 1.0; \) smooth concrete
\( 1.2; \) worn concrete, brick, cold blacktop
\( 1.5; \) hot blacktop

University of Michigan

35. Rolling resistance coefficient for bias-ply tires on heavy trucks, decimal.

\[ f_{roll} = (0.0066 + 0.000046V)f \]

\( V = \) Velocity, ft/sec
\( f = \) Friction coefficient, decimal

\( 1.0; \) smooth concrete
\( 1.2; \) worn concrete, brick, cold blacktop
\( 1.5; \) hot blacktop

University of Michigan

**Acceleration / Deceleration Rate**

*Acceleration (positive)/Deceleration (negative) is the rate of change of velocity with respect to time*

1. Acceleration/deceleration rate per unit of time, \( \text{ft/sec}^2 \).

\[ a = fg \]

\( f = \) Accel / Decel factor, decimal
\( g = \) Gravitational constant, \( 32.2 \text{ ft/sec}^2 \)

2. Average acceleration/deceleration rate over a unit of time, \( \text{ft/sec}^2 \).

\[ a = 1.466S / T \]

\( S = \) Speed constant, mi/hr
\( T = \) Time, sec

3. Average acceleration/deceleration rate over a unit of time, \( \text{ft/sec}^2 \).

\[ a = V / T \]

\( V = \) Velocity constant, ft/sec
\( T = \) Time, sec
Stop, From or To

4. Acceleration/deceleration rate of an object from or to a stop knowing the mass and force applied, \( \text{ft/sec}^2 \).

\[
a = \frac{F}{m}
\]

- \( F \) = Force, lb
- \( m \) = Mass, lb-sec^2/ft

5. Acceleration/deceleration rate from or to a stop over a determined distance and a unit of time, \( \text{ft/sec}^2 \).

\[
a = \frac{d}{(0.5T^2)}
\]

- \( d \) = Distance, ft
- \( T \) = Time, sec

6. Acceleration/deceleration rate from or to a stop over a determined distance and a unit of time, \( \text{ft/sec}^2 \).

\[
a = \frac{2d}{T^2}
\]

- \( d \) = Distance, ft
- \( T \) = Time, sec

7. Acceleration/deceleration rate from or to a stop over a determined distance, \( \text{ft/sec}^2 \).

\[
a = \frac{V^2}{2d}
\]

- \( V \) = Velocity, ft/sec
- \( d \) = Distance, ft

Acceleration

8. Acceleration rate from one velocity to another over a unit of time, \( \text{ft/sec}^2 \).

\[
a = \frac{V_f - V_o}{T}
\]

- \( V_f \) = Velocity final, ft/sec
- \( V_o \) = Velocity initial, ft/sec
- \( T \) = Time, sec
9. Acceleration rate from one velocity to another over a determined distance, \( \text{ft/sec}^2 \).

\[
a = \frac{V_f^2 - V_o^2}{2d}
\]

- \( V_f \) = Velocity final, \( \text{ft/sec} \)
- \( V_o \) = Velocity initial, \( \text{ft/sec} \)
- \( d \) = Distance, \( \text{ft} \)

10. Acceleration rate over a determined distance and a unit of time, \( \text{ft/sec}^2 \).

\[
a = \frac{2d - V_o 2 T}{T^2}
\]

- \( V_o \) = Velocity initial, \( \text{ft/sec} \)
- \( d \) = Distance, \( \text{ft} \)
- \( T \) = Time, \( \text{sec} \)

Deceleration

11. Deceleration rate from one velocity to another over a unit of time, \( \text{ft/sec}^2 \).

\[
a = \frac{V_o - V_f}{T}
\]

- \( V_o \) = Velocity initial, \( \text{ft/sec} \)
- \( V_f \) = Velocity final, \( \text{ft/sec} \)
- \( T \) = Time, \( \text{sec} \)

12. Deceleration rate from one velocity to another over a determined distance, \( \text{ft/sec}^2 \).

\[
a = \frac{V_o^2 - V_f^2}{2d}
\]

- \( V_o \) = Velocity initial, \( \text{ft/sec} \)
- \( V_f \) = Velocity final, \( \text{ft/sec} \)
- \( d \) = Distance, \( \text{ft} \)
13. Deceleration rate over a determined distance and a unit of time, \( \text{ft/sec}^2 \).

\[
a = \frac{2(d - V_f T)}{T^2}
\]

\( d = \) Distance, \( \text{ft} \)
\( T = \) Time, \( \text{sec} \)

\( V_f = \) Velocity final, \( \text{ft/sec} \)

**Lateral**

14. Lateral acceleration rate of a vehicle negotiating a level curve at a determined velocity with a known radius at the center of mass, \( \text{ft/sec}^2 \).

\[
a_y = \frac{V^2}{r}
\]

\( V = \) Velocity, \( \text{ft/sec} \)

\( r = \) Radius traveled by center of mass, \( \text{ft} \)

15. Lateral acceleration rate of a vehicle negotiating a level curve at a determined velocity with an unknown radius at the center of mass, \( \text{ft/sec}^2 \).

\[
a_y = \frac{V^2}{(r - 0.5tw)}
\]

\( V = \) Velocity, \( \text{ft/sec} \)

\( r = \) Radius of yaw mark, \( \text{ft} \)

\( tw = \) Track width, \( \text{ft} \)

16. Acceleration factor in the \( x \)-direction, \( \text{decimal} \).

\[
a_x = \frac{F_{x \text{ max}}}{W}
\]

\( W = \) Weight of vehicle, \( \text{lb} \)

\( F_{x \text{ max}} = \) Maximum tractional force to which a vehicle can Produce
Aerodynamics

1. Dynamic pressure of the airflow at a given velocity, \( \text{lb-ft}^2 \).

\[ P_T = P_{\text{Static}} + P_{\text{Dynamic}} \]

2. Dynamic pressure of the airflow at a given velocity, \( \text{lb-ft}^2 \).

\[ P_T = P_{\text{Static}} + 0.5 \rho V^2 \]

\( \rho = \text{Mass density of air}, \text{ lb sec}^2/\text{ft}^4 \)  
(Eq #3)  
\( V = \text{Velocity of air relative to vehicle, ft/sec} \)  
\( P_{\text{Static}} = \text{Table 1} \)

3. Air density for any atmospheric condition, \( \text{lb sec}^2/\text{ft}^4 \). Temperatures must be converted to absolute units.

\[ \rho = 0.00236\left(\frac{P}{29.92}\right)\left(\frac{519}{(460 + T)}\right) \]

\( P = \text{Ambient pressure, in (Table 1)} \)  
\( T = \text{Air temperature, deg (Fahrenheit)} \)

4. Determine a calculated aerodynamic drag force, \( \text{lb} \).

\[ F_A = 0.5 \rho V^2 C_D A \]

\( \rho = \text{Mass density of air}, \text{ lb sec}^2/\text{ft}^4 \)  
(Eq #3)  
\( V = \text{Velocity of air relative to vehicle, ft/sec} \)  
\( C_D = \text{Aerodynamic drag coefficient, decimal} \)  
(Table 2a or 2b)  
\( A = \text{Vehicle frontal area, ft}^2 \)
5. Determine an aerodynamic drag coefficient, \textbf{decimal}.

\[ C_D = \frac{F_A}{0.5 \rho V^2 A} \]

\[ \rho = \text{Mass density of air, \ lb \ sec}^2/\text{ft}^4 \]

\[ V = \text{Velocity of air relative to vehicle, \ ft/sec} \]

\[ F_A = \text{Drag Force, \ lb (Eq \#4)} \]

\[ A = \text{Vehicle frontal area, \ ft}^2 \]

6. Horsepower required to move the vehicle against air resistance, \textbf{hp}.

\[ H_{pa} = \frac{F_A V_A}{550} \]

\[ F_A = \text{Drag Force, \ lb (Eq \#4)} \]

\[ V_A = \text{Air velocity over the vehicle, \ ft/sec} \]

7. Total rolling resistance of a vehicle proceeding down road, \textbf{lb}.

\[ F_R = W\left(f_{roll} \pm m\right) \]

\[ W = \text{Total static weight, \ lb} \]

\[ f_{roll} = \text{Rolling drag factor, \ decimal} \]

\[ m = \text{Slope, \ pct (maximum of 10\%)} \]

\[ (+ \text{if uphill, - if down hill}) \]

8. Horsepower required to move a vehicle against its rolling resistance, \textbf{hp}.

\[ H_{pr} = \frac{F_R V}{550} \]

\[ F_R = \text{Rolling resistance, \ lb (Eq \#7)} \]

\[ V = \text{Velocity of vehicle with respect to the road, \ ft/sec} \]

9. Determine the total horsepower, \textbf{hp}.

\[ H_p = H_{pa} + H_{pr} \]

\[ H_{pa} = \text{Air resistance, required horsepower, \ hp (Eq \#6)} \]

\[ H_{pr} = \text{Rolling resistance, required horsepower, \ hp (Eq \#8)} \]
10. Determine side forces in a constant wind, \textbf{lb}.

\[ F_s = 0.5 \rho V_w^2 C_s A \]

- \( \rho = \text{Mass density of air, lb sec}^{-2}/\text{ft}^{4} \)
- (Eq #3)
- \( V_w = \text{Total wind velocity, ft/sec} \)
- \( C_s = \text{Side force coefficient which is a function of relative wind angle, decimal} \)
- \( A = \text{Vehicle frontal area, ft}^{2} \)
  (Not the side area of the vehicle)

11. Yaw moment with side force winds, \textbf{ft-lb}.

\[ Y_m = 0.5 \rho V^2 C_{ym} A \ell \]

- \( V = \text{Velocity of vehicle, ft/sec} \)
- \( \rho = \text{Mass density of air, lb sec}^{-2}/\text{ft}^{4} \)
- (Eq #3)
- \( A = \text{Vehicle frontal area, ft}^{2} \)
- \( \ell = \text{Wheelbase, ft} \)
- \( C_{ym} = \text{Yaw moment coefficient, decimal} \)

12. Determine the aerodynamic drag force, \textbf{lb}.

\[ F_{AD} = 0.00115 C_D A V_r^2 \]

- \( C_D = \text{Aerodynamic drag coefficient, decimal} \)
  (Table 2a or 2b)
- \( V_r = \text{Relative velocity between vehicle and wind, ft/sec} \)
- \( A = \text{Vehicle frontal area, ft}^{2} \)
Wind Speed Required, Rollover

13. Theoretical wind speed required to cause wheel lift or rollover, \textbf{mi/hr}.

\[ S = \sqrt{\frac{0.5W tw}{0.0025614z}} \]

\( W = \) Gross weight of vehicle, \text{lb}  
\( tw = \) Track width, \text{ft}  
\( A = \) area of windward side, \text{ft}^2  
\( z = \) Vertical center of mass height, \text{ft} 

\textit{Ravensdale}  
\( A = \) area of windward side, \text{ft}^2  
\( z = \) Vertical center of mass height, \text{ft} 

14. Determine a lateral stability of a vehicle. The friction coefficient of the roadway must be greater than the value of the solution for the vehicle to rollover, \textbf{decimal}.

\[ f_y = \frac{tw}{2z} \]

\( tw = \) Track width, \text{in}  
\( z = \) Vertical center of mass height, \text{in} 

\textbf{Note}: Wind direction must be perpendicular to the vehicle. If the roadway friction is less than the vehicle’s calculated overturn friction coefficient (stability), the vehicle will slide rather than rollover.

15. Velocity from transmission measurements incorporating air resistance, \textbf{ft/sec}.

\[ V = \sqrt{\frac{i_T i_A n (Te / R) - f_{roll} W}{C_D A (\rho/2)}} \]

\( i_T = \) Transmission gear ratio, 00:1  
\( i_A = \) Axle ratio, 00:1  
\( n = \) Mechanical efficiency of drive train, decimal  
\( Te = \) Torque at maximum rpm, \text{ft/lb}  
\( R = \) Radius of drive wheel, \text{ft}  
\( f_{roll} = \) Rolling resistance coefficient, decimal  
\( W = \) Weight of vehicle, \text{lb}  
\( C_D = \) Aerodynamic drag coefficient, decimal  
\( A = \) Vehicle frontal area, \text{ft}^2  
\( \rho = \) Mass density of air, \text{lbsec}^2/\text{ft}^4
Airborne

Galileo Galilei (1564 – 1642)

1. Initial speed during a fall from a level take-off, \textbf{mi/hr}.

\[ S = 2.73d / \sqrt{h} \]

\( d = \text{Horizontal distance center of mass traveled from take-off to landing, ft} \)
\( h = \text{Vertical fall distance, ft} \)

2. Initial velocity during a fall from a level take-off, \textbf{ft/sec}.

\[ V = 4.01d / \sqrt{h} \]

\( d = \text{Horizontal distance center of mass traveled from take-off to landing, ft} \)
\( h = \text{Vertical fall distance, ft} \)

3. Speed required to flip at a 45° take-off with a level center of mass landing, \textbf{mi/hr}.

\[ S = 3.86\sqrt{d} \]

\( d = \text{Horizontal distance center of mass traveled from take-off to landing, ft} \)

4. Velocity required to flip at a 45° take-off with a level center of mass landing, \textbf{ft/sec}.

\[ V = d\sqrt{g / d} \]

\( d = \text{Horizontal distance center of mass traveled from take-off to landing, ft} \)
\( g = \text{Gravitational constant, 32.2 ft/sec}^2 \)
5. Speed required to vault with a grade less than 6.8°, \textbf{mi/hr}.

\[ S = \frac{2.73d}{ \sqrt{dm - h} } \]

\( d = \) Horizontal distance center of mass traveled from take-off to landing, \text{ ft}

\( h = \) Vertical fall distance, \text{ ft}

(negative value (-) for a lower center of mass landing)

\( m = \) Grade, maximum 6.8°, decimal

(negative value (-) for decline)

6. Velocity required to vault with a grade less than 6.8°, \textbf{ft/sec}.

\[ V = \frac{4.01d}{ \sqrt{dm - h} } \]

\( d = \) Horizontal distance center of mass traveled from take-off to landing, \text{ ft}

\( h = \) Vertical fall distance, \text{ ft}

(negative value (-) for a lower center of mass landing)

\( m = \) Grade, maximum 6.8°, decimal

(negative value (-) for decline)

7. Speed required to vault or flip with a 45° take-off, \textbf{mi/hr}.

\[ S = \frac{3.86d}{ \sqrt{d - h} } \]

\( d = \) Horizontal distance center of mass traveled from take-off to landing, \text{ ft}

\( h = \) Vertical distance from the plane of take-off to landing, \text{ ft}

(negative value (-) for a lower center of mass landing)
8. Speed required to vault with a grade greater than 6.8°, **mi/hr**.

\[ S = \frac{2.73d}{\sqrt{d \cos \theta \sin \theta - h \cos^2 \theta}} \]

\( d \) = Horizontal distance center of mass traveled from take-off to landing, ft

\( h \) = Vertical distance from the plane of take-off to landing, ft (negative value (-) for a lower center of mass landing)

\( \theta \) = Angle of grade, deg

9. Speed required to vault with an angle of take-off exceeding 6.8°, **mi/hr**.

\[ S = \frac{2.73d}{\cos \theta \sqrt{d \tan \theta - h}} \]

\( d \) = Horizontal distance center of mass traveled from take-off to landing, ft

\( h \) = Vertical distance from the plane of take-off to landing, ft (negative value (-) for a lower center of mass landing)

\( \theta \) = Angle of take-off, deg

10. Velocity required to vault with an angle of take-off exceeding 6.8°, **ft/sec**.

\[ V = \frac{4.01d}{\cos \theta \sqrt{d \tan \theta - h}} \]

\( d \) = Horizontal distance center of mass traveled from take-off to landing, ft

\( h \) = Vertical distance from the plane of take-off to landing, ft (negative value (-) for a lower center of mass landing)

\( \theta \) = Angle of take-off, deg
11. Velocity required to fall with a grade less than 6.8°, \( \text{ft/sec} \).

\[ V = d \sqrt{\frac{g}{2(dm - h)}} \]

\( d \) = Horizontal distance center of mass traveled from take-off to landing, \( \text{ft} \)

\( h \) = Vertical fall distance, \( \text{ft} \) (negative value (-) for a lower center of mass landing)

\( g \) = Gravitational constant, 32.2 \( \text{ft/sec}^2 \)

\( m \) = Grade, maximum 6.8°, decimal

(negative value (-) for decline)

12. Velocity required to vault or flip with a 45° take-off angle, \( \text{ft/sec} \).

\[ V = d \sqrt{\frac{g}{(d - h)}} \]

\( d \) = Horizontal distance center of mass traveled from take-off to landing, \( \text{ft} \)

\( h \) = Vertical distance from the plane of take-off to landing, \( \text{ft} \) (negative value (-) for a lower center of mass landing)

\( g \) = Gravitational constant, 32.2 \( \text{ft/sec}^2 \)

13. Velocity required to vault with an angle of take-off exceeding 6.8°, \( \text{ft/sec} \).

\[ V = \frac{4.01d}{\sqrt{d \cos \theta \sin \theta - h \cos^2 \theta}} \]

\( d \) = Horizontal distance center of mass traveled from take-off to landing, \( \text{ft} \)

\( h \) = Vertical distance from the plane of take-off to landing, \( \text{ft} \) (negative value (-) for a lower center of mass landing)

\( \theta \) = Angle of take-off, \( \text{deg} \)
14. Velocity required to vault with an angle of take-off exceeding 6.8°, \textbf{ft/sec}.

\[ V = \sqrt{\frac{d^2 g}{2} \left( \cos^2 \theta (d \tan \theta - h) \right)} \]

\( d = \) Horizontal distance center of mass traveled from take-off to landing, \text{ ft} \\
\( h = \) Vertical distance from the plane of take-off to landing, \text{ ft} (negative value (-) for a lower center of mass landing) \\
\( \theta = \) Angle of take-off, \text{ deg} \\
\( g = \) Gravitational constant, \(32.2 \text{ ft/sec}^2\)

15. Velocity required to vault with an angle of take-off exceeding 6.8°, \textbf{ft/sec}.

\[ V = d \sqrt{\frac{g}{2 \cos \theta (d \sin \theta - h \cos \theta)}} \]

\( d = \) Horizontal distance center of mass traveled from take-off to landing, \text{ ft} \\
\( h = \) Vertical distance from the plane of take-off to landing, \text{ ft} (negative value (-) for a lower center of mass landing) \\
\( \theta = \) Angle of take-off, \text{ deg} \\
\( g = \) Gravitational constant, \(32.2 \text{ ft/sec}^2\)

16. Velocity at landing from a fall with a level take-off, \textbf{ft/sec}.

\[ V_L = \sqrt{2gh} \]

\( h = \) Vertical fall distance, \text{ ft} \\
\( g = \) Gravitational constant, \(32.2 \text{ ft/sec}^2\)

17. Velocity at landing from a vault, \textbf{ft/sec}.

\[ V_{Lb} = \sqrt{V_h^2 + V_f^2} \]

\( V_h = \) Initial horizontal velocity prior to take-off slope, \text{ ft/sec} (Eq #18) \\
\( V_f = \) Final vertical velocity, \text{ ft/sec} (Eq #20)
18. Initial horizontal velocity prior to the take-off slope, \textbf{ft/sec}.

\[ V_h = V \cos \theta \]

\( V = \text{Velocity, ft/sec (Eq #10 thru 15)} \)

\( \theta = \text{Angle of take-off, deg} \)

19. Initial vertical velocity during the take-off, \textbf{ft/sec}.

\[ V_v = V \sin \theta \]

\( V = \text{Velocity, ft/sec (Eq #10 thru 15)} \)

\( \theta = \text{Angle of take-off, deg} \)

20. Final vertical velocity, \textbf{ft/sec}.

\[ V_f = \sqrt{(V \sin \theta)^2 + 2gh_z} \]

\( V = \text{Velocity, ft/sec} \)

\( h_z = \text{Maximum vertical height to landing, ft (Eq #31)} \)

\( \theta = \text{Angle of take-off, deg} \)

\( g = \text{Gravitational constant, 32.2 ft/sec} \)

21. Optimum angle of take-off during a flip or vault to determine minimum required speed, \textbf{deg}.

\[ \theta = 0.5 \cos^{-1}\left(\frac{-h}{\sqrt{d^2 + h^2}}\right) \]

\( d = \text{Horizontal distance center of mass traveled from take-off to landing, ft} \)

\( h = \text{Vertical distance from the plane of take-off to landing, ft} \) (negative value (-) for a lower center of mass landing)
22. Optimum angle of take-off during a flip or vault to determine minimum required speed, deg. Add 90° if the solution is negative.

\[ \theta = 0.5 \tan^{-1} \left( -\frac{d}{h} \right) \]

\( d \) = Horizontal distance center of mass traveled from take-off to landing, ft
\( h \) = Vertical distance from the plane of take-off to landing, ft (negative value (-) for a lower center of mass landing)

23. Time of flight to a particular point along the trajectory, sec.

\[ T = \frac{d}{V \cos \theta} \]

\( d \) = Horizontal distance from take-off to the specific point along the trajectory, ft
\( V \) = Initial take-off velocity, ft/sec
\( \theta \) = Angle of take-off, deg

24. Time of flight, sec.

\[ T = \left( \frac{V_v + \sqrt{V_v^2 - h_2g}}{g} \right) \]

\( V_v \) = Initial vertical velocity, ft/sec (Eq #19)
\( h \) = Vertical distance from the plane of take-off to landing, ft (negative value (-) for a lower center of mass landing)
\( g \) = Gravitational constant, 32.2 ft/sec²
25. Time of flight, \textbf{sec}.

\[ T = \frac{-V \sin \theta - \sqrt{(V \sin \theta)^2 - h^2g}}{-g} \]

\(V\) = Velocity, \textit{ft/sec}\n\(h\) = Vertical distance from the plane of take-off to landing, \textit{ft} (negative value (-) for a lower center of mass landing)\n\(\theta\) = Angle of take-off, \textit{deg}\n\(g\) = Gravitational constant, 32.2 ft/sec\(^2\)

26. Time of flight from take-off to maximum vertical height, \textbf{sec}.

\[ T_m = \frac{V \sin \theta}{g} \]

\(V\) = Velocity, \textit{ft/sec}\n\(\theta\) = Angle of take-off, \textit{deg}\n\(g\) = Gravitational constant, 32.2 ft/sec\(^2\)

27. Maximum vertical height reached above the plane of take-off, \textbf{ft}.

\[ h_m = T_m V \sin \theta - 0.5gT_m^2 \]

\(V\) = Velocity, \textit{ft/sec}\n\(T_m\) = Time to maximum vertical height, \textit{sec} (Eq #26)\n\(\theta\) = Angle of take-off, \textit{deg}\n\(g\) = Gravitational constant, 32.2 ft/sec\(^2\)

28. Maximum vertical height reached above the plane of take-off, \textbf{ft}.

\[ h_m = \frac{(V \sin \theta)^2}{2g} \]

\(V\) = Velocity, \textit{ft/sec}\n\(\theta\) = Angle of take-off, \textit{deg}\n\(g\) = Gravitational constant, 32.2 ft/sec\(^2\)
29. Maximum vertical height reached above the plane of take-off, \( \text{ft} \).

\[
h_m = \frac{V_v^2}{2g}
\]

\( V_v \) = Initial vertical velocity, \( \text{ft/sec} \) (Eq #19)
\( g \) = Gravitational constant, 32.2 \( \text{ft/sec}^2 \)

30. Maximum vertical height reached above the plane of take-off, \( \text{ft} \).

\[
h_m = 0.033S^2 Sin^2 \theta
\]

\( S \) = Speed, \( \text{mi/hr} \)
\( \theta \) = Angle of take-off, \( \text{deg} \)

31. Vertical distance from maximum height to landing, \( \text{ft} \).

\[
h_2 = h_m - h
\]

\( h_m \) = Maximum vertical height above the plane of take-off, \( \text{ft} \) (Eq #25 thru 27)
\( h \) = Vertical distance from the plane of take-off to landing, \( \text{ft} \) (negative value (-) for a lower center of mass landing)

32. Time from maximum vertical height to landing for a vehicle, which has gone airborne, \( \text{sec} \).

\[
T_L = \sqrt{\frac{2h_2}{g}}
\]

\( h_2 \) = Distance from maximum vertical height to landing, \( \text{ft} \) (Eq #31)
\( g \) = Gravitational constant, 32.2 \( \text{ft/sec}^2 \)
33. Horizontal distance from the take-off to a point perpendicular to the maximum vertical height, \( \text{ft} \).

\[
d_m = \frac{\tan \theta (V \cos \theta)^2}{g}
\]

\( \theta \) = Angle of take-off, \( \deg \)
\( g \) = Gravitational constant, \( 32.2 \text{ ft/sec}^2 \)

34. Distance traveled during flight from take-off to the maximum vertical height, \( \text{ft} \).

\[
d_a = \frac{V^2 \sin \theta \cos \theta}{g}
\]

\( \theta \) = Angle of take-off, \( \deg \)
\( g \) = Gravitational constant, \( 32.2 \text{ ft/sec}^2 \)

35. Vertical fall distance from any point along the trajectory measured along a horizontal plane, \( \text{ft} \). (negative value (-) for a lower center of mass landing)

\[
h = Tan \theta d - 0.5gd^2 / (V \cos \theta)^2
\]

\( V \) = Initial take-off velocity, \( \text{ft/sec} \)
\( d \) = Horizontal distance from take-off to the specific point along the trajectory, \( \text{ft} \)
\( \theta \) = Angle of take-off, \( \deg \)
\( g \) = Gravitational constant, \( 32.2 \text{ ft/sec}^2 \)

36. Vertical fall distance from the take-off point to the landing, \( \text{ft} \). (negative value (-) for a lower center of mass landing)

\[
h = V \sin \theta T - 0.5gT^2
\]

\( V \) = Initial take-off velocity, \( \text{ft/sec} \)
\( T \) = Time of flight, \( \text{sec} \) (Eq #24, 25)
\( \theta \) = Angle of take-off, \( \deg \)
\( g \) = Gravitational constant, \( 32.2 \text{ ft/sec}^2 \)
37. Horizontal distance traveled from a level take-off to landing knowing the initial velocity and vertical fall distance, ft.

\[ d = V \sqrt{\frac{2h}{g}} \]

\( V \) = Velocity, ft/sec
\( h \) = Vertical fall distance, ft
\( g \) = Gravitational constant, 32.2 ft/sec^2

38. Horizontal distance traveled from take-off to landing knowing the initial velocity and angle at take-off, ft.

\[ d = V \cos \theta T \]

\( V \) = Velocity, ft/sec
\( T \) = Time of flight, sec (Eq #24, 25)
\( \theta \) = Angle of take-off, deg

39. Maximum flight distance with a 45-degree take-off if initial velocity is known, ft.

\[ d = \frac{V_0^2}{g} \]

\( V_0 \) = Velocity original, ft/sec
\( g \) = Gravitational constant, 32.2 ft/sec^2

**Audible**

**Audible Levels**

<table>
<thead>
<tr>
<th>Source</th>
<th>dBA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distinguishable</td>
<td>9 – 10 dBA above threshold</td>
</tr>
<tr>
<td>Insertion loss for vehicle</td>
<td>30 dBA</td>
</tr>
<tr>
<td>Inside Vehicle: 50 mph ~ Average</td>
<td>72 dBA</td>
</tr>
<tr>
<td>Windows closed / no radio</td>
<td></td>
</tr>
<tr>
<td>Horn from Locomotive: 100 ft</td>
<td>96 dBA 49CFR229</td>
</tr>
<tr>
<td>Interior vehicle ~ Average</td>
<td>80 – 90 dBA</td>
</tr>
<tr>
<td>Interior cab of Truck ~ Average</td>
<td>85 + dBA</td>
</tr>
</tbody>
</table>

*Train Accident Reconstruction p.228 ~ Loumiet*
### Example Findings

<table>
<thead>
<tr>
<th>Time Before Impact</th>
<th>A Outside dBA Level</th>
<th>B Insertion dBA Loss</th>
<th>C Signal dBA Inside</th>
<th>D Operating dBA Level</th>
<th>E Signal-to-Noise Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 Seconds</td>
<td>92</td>
<td>30</td>
<td>62</td>
<td>73</td>
<td>-11</td>
</tr>
<tr>
<td>4 Seconds</td>
<td>102</td>
<td>30</td>
<td>72</td>
<td>73</td>
<td>-1</td>
</tr>
<tr>
<td>3 Seconds</td>
<td>100</td>
<td>30</td>
<td>70</td>
<td>73</td>
<td>-3</td>
</tr>
<tr>
<td>2.5 Seconds</td>
<td>103</td>
<td>30</td>
<td>73</td>
<td>73</td>
<td>0</td>
</tr>
<tr>
<td>2 Second</td>
<td>104.5</td>
<td>30</td>
<td>74.5</td>
<td>73</td>
<td>1.5</td>
</tr>
<tr>
<td>1.5 Seconds</td>
<td>107</td>
<td>30</td>
<td>77</td>
<td>73</td>
<td>4</td>
</tr>
<tr>
<td>1 Second</td>
<td>111.5</td>
<td>30</td>
<td>81.5</td>
<td>73</td>
<td>8.5</td>
</tr>
</tbody>
</table>

*Train Accident Reconstruction p.233 ~ Loumiet*

### Bicycle

A vehicle consisting of a light frame mounted on two wire-spoked wheels one behind the other and having a seat, handlebars for steering, brakes, and two pedals or a small motor by which it is driven.

### Acceleration Rate

Over distance of 40 feet

<table>
<thead>
<tr>
<th>Speed</th>
<th>Men</th>
<th>Women</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slow</td>
<td>6.54 sec</td>
<td>5.84 sec</td>
</tr>
<tr>
<td>Fast</td>
<td>3.97 sec</td>
<td>3.95 sec</td>
</tr>
<tr>
<td>Average</td>
<td>4.90 sec</td>
<td>5.02 sec</td>
</tr>
<tr>
<td>Acceleration rate</td>
<td>3.31 ft/sec</td>
<td>3.17 ft/sec</td>
</tr>
<tr>
<td>Acceleration factor</td>
<td>0.10</td>
<td>0.10</td>
</tr>
</tbody>
</table>

*Haight*

### Revolutions per Minute

<table>
<thead>
<tr>
<th>Normal Riding</th>
<th>Serious Exercising</th>
<th>Racing</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 – 80 rpm</td>
<td>100 – 125 rpm</td>
<td>140 – 150 rpm</td>
</tr>
</tbody>
</table>
Lean Angle

<table>
<thead>
<tr>
<th></th>
<th>Average</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12 - 15°</td>
<td>15 - 20°</td>
</tr>
</tbody>
</table>

Lateral Acceleration Factor

Maximum = 0.25 to 0.30

1. Calculate the gear inch knowing the number of teeth on the chain wheel and on the freewheel, \text{in}.

\[ G_i = \frac{C_n}{F_n} W_n \pi \]

\( C_n \) = Number of teeth on chain wheel, \# \n
\( F_n \) = Number of teeth on freewheel, \# \n
\( W_n \) = Diameter of rear wheel, \text{in} \n
\( \pi \) = Pi, 3.141592654

2. Riding velocity knowing gear inch and pedal revolutions per minute, \text{ft/sec}.

\[ V = G_i R_p 0.00139 \]

\( G_i \) = Gear inch (Eq #1), \text{in} \n
\( R_p \) = Pedal revolutions per minute, \text{rpm}

3. Speed of vehicle striking a bicyclist, \text{ft/sec}.

NOTE: Adult & Child Pontoon Vehicles and Adult V-Contour Vehicles equation

\[ V = \left( -\frac{a^2 \times d}{2} + \sqrt{\left(\frac{a \times c}{3}\right)^3 + \left(\frac{a^2 \times d}{2}\right)^2} \right)^{\frac{1}{3}} + \left( -\frac{a^2 \times d}{2} - \sqrt{\left(\frac{a \times c}{3}\right)^3 + \left(\frac{a^2 \times d}{2}\right)^2} \right)^{\frac{1}{3}} \]

\[ V = \frac{a}{a} \]

Sturtz

Child V-Contour Vehicles

\[ V = \sqrt{\frac{d_i - h}{a}} \]

Box Vehicles (Forward Projections)

\[ V = \frac{d_i - h}{0.779} \]
User Inputs
- $d_t$: Throw Distance
- $h$: Height of pedestrian's Center of Mass
- $d = h - d_t$
- $a$ & $c$: Equation Constants

Solved in:
- fps or m/s

Miscellaneous
Value of $\mu$ ranged from 0.4 to 0.71

**V-Contour Vehicles** - Low pointed front-end vehicles
**Pontoon Vehicles** - Traditional style front end vehicles.
**Box Vehicle** equation (forward projection) quickly becomes unstable as the throw distance exceeds 50 feet/915 meters.

<table>
<thead>
<tr>
<th>Constant values</th>
<th>$a$ - Imperial</th>
<th>$a$ - Metric</th>
<th>$c$-value</th>
<th>$d$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pontoon Vehicles (adult)</td>
<td>0.0001672</td>
<td>0.00182</td>
<td>0.61</td>
<td></td>
</tr>
<tr>
<td>V-Contour Vehicles (adult)</td>
<td>0.00145</td>
<td>0.0001347</td>
<td>0.645</td>
<td></td>
</tr>
<tr>
<td>Pontoon Vehicles (child)</td>
<td>0.0021</td>
<td>0.0001951</td>
<td>0.76</td>
<td></td>
</tr>
<tr>
<td>V-Contour Vehicles (child)</td>
<td>0.02027</td>
<td>0.0665</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>Bicycle</td>
<td>.0002900</td>
<td>.0369</td>
<td>.15/3.25333 ped-com – d</td>
<td></td>
</tr>
</tbody>
</table>
## Braking Efficiency

Table 1: Vehicle Braking Percentage (forward frontal heading)

<table>
<thead>
<tr>
<th>Vehicle Type</th>
<th>Percentage of Braking (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Passenger Vehicles</strong></td>
<td></td>
</tr>
<tr>
<td>Only front wheels locked</td>
<td>60-70</td>
</tr>
<tr>
<td>Only rear wheels locked</td>
<td>30-40</td>
</tr>
<tr>
<td>ABS equipped; full braking</td>
<td>100+</td>
</tr>
<tr>
<td><strong>Motorcycles</strong></td>
<td></td>
</tr>
<tr>
<td>Free rolling</td>
<td>.01-.02</td>
</tr>
<tr>
<td>Front/Rear Full Lockup</td>
<td>.80-1.1</td>
</tr>
<tr>
<td>Moderate/heavy front brake application with rear wheel lockup</td>
<td>(f_e = (\mu / 2 + \mu) / 2)</td>
</tr>
<tr>
<td><strong>Front Wheel Only</strong></td>
<td></td>
</tr>
<tr>
<td>Clean, dry surface</td>
<td>.65-.70</td>
</tr>
<tr>
<td><strong>Rear Wheel Only</strong></td>
<td></td>
</tr>
<tr>
<td>Clean, dry surface</td>
<td>.35-.45</td>
</tr>
<tr>
<td>Soft soil, sand</td>
<td>.90-1.2</td>
</tr>
<tr>
<td>Hard soil</td>
<td>.70</td>
</tr>
</tbody>
</table>

Proper Brake Adjustment For The Following Values Apply

<table>
<thead>
<tr>
<th>Vehicle Type</th>
<th>Percentage of Braking (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Straight Trucks</strong></td>
<td>70-80</td>
</tr>
<tr>
<td><strong>Loaded Tractor/Semi Trailer (5 axle)</strong></td>
<td>60-75</td>
</tr>
<tr>
<td>(10% steer, 36% drives, 24% trailer)</td>
<td></td>
</tr>
<tr>
<td><strong>Doubles (Cab over Engine tractor &amp; twin 28's)</strong></td>
<td>80</td>
</tr>
<tr>
<td><strong>Dump Trucks</strong></td>
<td>70-80</td>
</tr>
<tr>
<td><strong>Concrete Mixers (Caution: Limited Testing)</strong></td>
<td>45-70</td>
</tr>
<tr>
<td><strong>Motor Homes (Caution: Limited Testing)</strong></td>
<td>70-80</td>
</tr>
</tbody>
</table>
### Commercial Buses

<table>
<thead>
<tr>
<th>Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>MC-9 (Greyhound style)</td>
<td>70-80</td>
</tr>
<tr>
<td>Transit (including articulated city buses)</td>
<td>70-85</td>
</tr>
<tr>
<td>School</td>
<td>70-80</td>
</tr>
</tbody>
</table>

### Bobtails

<table>
<thead>
<tr>
<th>Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cab over Engine</td>
<td>30</td>
</tr>
<tr>
<td>Conventional</td>
<td>35</td>
</tr>
</tbody>
</table>

*Note: Front axle brakes may slightly increase the braking coefficient. However, the coefficient will still fall within the range of 0.3 - 0.4*

**CAUTION:** If the vehicle is equipped with a brake proportioning valve, the percentage will increase dramatically to 80-85%.

- Bobtails w/ BP-1 & BP-2 values: 80-85
- 'Anteaters' w/BP-1: 92
- Frontlines (86+) w/WABCO 6 Channel anti-lock: 84-87
  (Westinghouse Air Brake Company)

---

1. Percentage of braking applied during a deceleration with a known friction coefficient for the surface and a deceleration factor for the vehicle, **pct**.

   \[ n = \left( \frac{f}{\mu} \right) \times 100 \]

   - \(n\) = Deceleration factor, decimal
   - \(f\) = Deceleration factor, decimal
   - \(\mu\) = Friction coefficient, decimal

2. Percentage of braking applied during a deceleration to a stop, **decimal**.

   \[ n = \frac{S^2}{(30d\mu)} \]

   - \(n\) = Deceleration factor, decimal
   - \(S\) = Speed, mi/hr
   - \(d\) = Distance, ft
   - \(\mu\) = Friction coefficient, decimal
3. Percentage of braking applied during a deceleration to a stop, decimal.

\[ n = \frac{V^2}{(2gd\mu)} \]

\( V = \) Velocity, \( \text{ft/sec} \)
\( d = \) Distance, \( \text{ft} \)
\( \mu = \) Friction coefficient, decimal
\( g = \) Gravitational constant, \( 32.2 \text{ ft/sec}^2 \)

**Trailer, Equivalent Deceleration Factor**

4. Equivalent deceleration factor for a vehicle/trailer combination with no braking of the trailer, decimal.

\[ f_e = f\frac{W_v}{W_v + W_t} \]

\( f = \) Vehicle deceleration factor, decimal
\( W_v = \) Static weight of vehicle, lb
\( W_t = \) Static weight of trailer, lb

5. Braking force applied to a tire, which is at its frictional limit during a cornering maneuver, lb.

\[ F_{sh} = W\mu\sin\alpha \]

\( \mu = \) Friction coefficient, decimal
\( \alpha = \) Tire slip angle, deg
\( W = \) Weight on tire, lb

**Brake Lag**

6. Velocity at commencement of brake activation incorporating the time of brake lag, ft/sec.

Utilize for standard, hydraulic fluid transfer brake systems only.

\[ V_b = V + 0.6aT_b \]

\( V = \) Initial velocity calculated, \( \text{ft/sec} \)
\( a = \) Deceleration rate, \( \text{ft/sec}^2 \)
\( T_b = \) Brake lag time, sec

Recommended brake lag time of 0.3 - 0.55 seconds for standard brake systems.
7. Distance traveled during brake lag time, **ft**.

\[ d_b = VT_b - 0.5gf 0.6T_b^2 \]

- **V** = Initial velocity calculated, **ft/sec**
- **f** = Deceleration factor, **decimal**
- **T_b** = Brake lag time, **sec**

Recommended brake lag time of 0.3 - 0.55 seconds for standard brake systems.

- **g** = Gravitational constant, 32.2 **ft/sec**

8. Velocity at commencement of brake activation incorporating distance traveled during brake lag time, **ft/sec**. Utilize for standard, hydraulic fluid transfer brake systems only.

\[ V_b = \sqrt{V^2 - 2gf 0.6d_b} \]

- **d_b** = Brake lag distance, **ft** (Eq # 7)
- **V** = Initial velocity calculated, **ft/sec**
- **f** = Deceleration factor, **decimal**
- **g** = Gravitational constant, 32.2 **ft/sec**

9. Distance traveled at commencement of brake activation incorporating distance traveled during brake lag time, **ft**. Utilize for standard, hydraulic fluid transfer brake systems only.

\[ d = \frac{(V^2 - 2gf 0.6d_b)}{2fg} \]

- **d_b** = Brake lag distance, **ft** (Eq # 7)
- **V** = Initial velocity calculated, **ft/sec**
- **f** = Deceleration factor, **decimal**
- **g** = Gravitational constant, 32.2 **ft/sec**


Center of Mass

The point in a system of bodies at which the mass of the system may be considered to be concentrated and at which external forces may be considered to be applied. Also called barycenter, centroid.

Table 1: Rule of Thumb

<table>
<thead>
<tr>
<th>Thumb</th>
<th>Cars</th>
<th>Trucks</th>
</tr>
</thead>
<tbody>
<tr>
<td>CM height = 21.0 in</td>
<td>21.29 in ± 1.5 in</td>
<td>26.71 in ± 4.0 in</td>
</tr>
<tr>
<td>CM height = 40% Roof Hgt</td>
<td>39.5% ± 2.6 %</td>
<td>38.7% ± 3.5 %</td>
</tr>
</tbody>
</table>

- CM for BobTail snubnose semi tractor is about 40-50 inches from ground
- U.S. Federal regulations do not permit CM over 75 inches
- CM for pedestrian can be estimated in three ways:
  - at the iliac crest (Spitz)
  - third lumbar vertebrae (Snyder & Hermance)
  - 57% of the pedestrian’s height (Wood)

Table 2: Inertial Parameters

<table>
<thead>
<tr>
<th>Thumb</th>
<th>Cars</th>
<th>Trucks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{pitch}$ ft–lb–sec$^2$</td>
<td>0.99W-1149</td>
<td>1.12W-1657</td>
</tr>
<tr>
<td>$I_{roll}$ ft–lb–sec$^2$</td>
<td>0.18W-150</td>
<td>0.22W-235</td>
</tr>
<tr>
<td>$I_{yaw}$ ft–lb–sec$^2$</td>
<td>1.03W-1206</td>
<td>1.03W-1343</td>
</tr>
</tbody>
</table>

NHTSA

Longitudinal Center of Mass

1. Longitudinal center of mass measured from the front axle, $f_t$.

$$x_f = \frac{W_R \ell}{W}$$

$\ell$ = Wheelbase, ft

$W_R$ = Static rear axle weight, lb

$W$ = Total static weight, lb
2. Longitudinal center of mass measured from the front axle, \( \text{ft} \).

\[
x_F = (1 - W_{Fi}) \ell
\]

\( \ell \) = Wheelbase, \( \text{ft} \)

\( W_{Fi} \) = Fraction of weight on front wheels, decimal \( (W_F / W) \)

3. Longitudinal center of mass measured from the front axle as a decimal fraction of the wheelbase, \text{decimal}.

\[
x_{Fi} = x_F / \ell
\]

\( x_F \) = Longitudinal center of mass from the front axle, \( \text{ft} \) (Eq #1)

\( \ell \) = Wheelbase, \( \text{ft} \)

4. Longitudinal center of mass measured from the rear axle, \( \text{ft} \).

\[
x_R = \frac{W_F \ell}{W}
\]

\( \ell \) = Wheelbase, \( \text{ft} \)

\( W_F \) = Static front axle weight, \( \text{lb} \)

\( W \) = Total static weight, \( \text{lb} \)

5. Longitudinal center of mass measured from the rear axle, \( \text{ft} \).

\[
x_R = (1 - W_{Ri}) \ell
\]

\( \ell \) = Wheelbase, \( \text{ft} \)

\( W_{Ri} \) = Fraction of weight on rear wheels, decimal \( (W_R / W) \)

6. Longitudinal center of mass measured from the rear axle as a decimal fraction of the wheelbase, \text{decimal}.

\[
x_{Ri} = x_R / \ell
\]

\( x_R \) = Longitudinal center of mass from the rear axle, \( \text{ft} \) (Eq #4, 5)

\( \ell \) = Wheelbase, \( \text{ft} \)
Lateral Center of Mass

7. Lateral center of mass measured from the left side, ft.

\[ y_l = \frac{W_{tol}}{W} \]

\( tw = \) Track width, ft
\( W_r = \) Static right side weight, lb
\( W = \) Total static weight, lb

8. Lateral center of mass measured from the right side, ft.

\[ y_r = \frac{W_{tow}}{W} \]

\( tw = \) Track width, ft
\( W_l = \) Static left side weight, lb
\( W = \) Total static weight, lb

Vertical Center of Mass


\[ z = \frac{(W_h - W_F) \ell \sqrt{\ell^2 - (h - r)^2}}{W (h - r)} + r \]

\( \ell = \) Wheelbase, ft
\( h = \) Vertical height rear axle elevated, ft (1/3 of wheelbase)
\( r = \) Radius of drive wheels, ft
\( W_h = \) Front axle weight, rear elevated, lb
\( W_F = \) Static front axle weight, lb
\( W = \) Total static weight, lb
10. Vertical center of mass height, \( \text{ft} \). Front elevated.

\[
z = \frac{(W_h - W_R) \ell}{W (h - r) + r} \sqrt{\frac{\ell^2 - (h - r)^2}{h - r}}
\]

- \( \ell \) = Wheelbase, \( \text{ft} \)
- \( h \) = Vertical height front axle elevated, \( \text{ft} \)  
  \( \text{(1/3 of wheelbase)} \)
- \( r \) = Radius of drive wheels, \( \text{ft} \)
- \( W_h \) = Rear axle weight, front elevated, \( \text{lb} \)
- \( W_R \) = Static rear axle weight, \( \text{lb} \)
- \( W \) = Total static weight, \( \text{lb} \)

11. Vertical center of mass height as a decimal fraction of the wheelbase, \textbf{decimal}.

\[
z_i = \frac{z}{\ell}
\]

- \( z \) = Vertical center of mass height, \( \text{ft} \)  
  \( \text{(Eq #9, 10)} \)
- \( \ell \) = Wheelbase, \( \text{ft} \)

**Trailer, Center of Mass**

12. Longitudinal center of mass of combined trailer with load measured from a datum line, \( \text{ft} \).

\[
x = \frac{W_L x_L + W_T x_T}{W}
\]

- \( x_L \) = Longitudinal distance from the datum line to  
  center of mass of load, \( \text{ft} \)
- \( x_T \) = Longitudinal distance from the datum line to  
  the trailer’s center of mass, \( \text{ft} \)
- \( W_L \) = Static weight of load, \( \text{lb} \)
- \( W_T \) = Static weight of trailer, \( \text{lb} \)
- \( W \) = Total static weight of semi trailer and load, \( \text{lb} \)
13. Lateral center of mass of combined trailer with load measured from a datum line, \text{ft}.

\[
y = \frac{W_L y_L + W_T y_T}{W}
\]

\(y_L\) = Lateral distance from the datum line to center of mass of load, \text{ft}
\(y_T\) = Lateral distance from the datum line to the trailer’s center of mass, \text{ft}
\(W_L\) = Static weight of load, \text{lb}
\(W_T\) = Static weight of trailer, \text{lb}
\(W\) = Total static weight of semi trailer and load, \text{lb}

14. Vertical center of mass height of combined trailer with load, \text{ft}.

\[
z = \frac{W_L z_L + W_T z_T}{W}
\]

\(z_L\) = Vertical center of mass height of load from the ground, \text{ft}
\(z_T\) = Vertical center of mass height of trailer, \text{ft}
\(W_L\) = Static weight of load, \text{lb}
\(W_T\) = Static weight of trailer, \text{lb}
\(W\) = Total static weight of semi trailer and load, \text{lb}

**Collinear Avoidance (Stationary Hazard)**

*Maximum Speed/Velocity*

1. Maximum speed possible in order to stop from a known distance; (hill crest, bend in roadway) when first perception of an obstacle occurs, \text{mi/hr}.

\[
S = 21.96 f \left[ \sqrt{\frac{T^2}{f^2} + 0.0621 d} / f - T \right]
\]

\(f\) = Deceleration factor, decimal
\(d\) = Total distance to Impact, \text{ft}
\{including P/R distance\}
\(T\) = Perception/Reaction time, sec
2. Maximum velocity possible in order to stop from a known distance; (hill crest, bend in roadway) when first perception of an obstacle occurs, **ft/sec**.

\[ V = f g \sqrt{\frac{T^2 + 2d}{fg} - T} \]

\( f = \text{Deceleration factor, decimal} \)
\( d = \text{Total distance to Impact, ft} \)
\( T = \text{Perception/Reaction time, sec} \)
\( g = 32.2 \text{ ft/sec}^2 \)

**Reasonable & Prudent Speed**

3. Reasonable and prudent speed under adverse conditions knowing the speed limit and the friction coefficient for the normal and adverse conditions, **mi/hr**.

\[ S_R = \frac{\sqrt{S_L^2 f_a}}{f_n} \]

\( S_R = \text{Posted speed limit, mi/hr} \)
\( f_a = \text{Friction coefficient for adverse conditions, decimal} \)
\( f_n = \text{Friction coefficient for normal conditions, decimal} \)

**Original Speed**

4. Original speed knowing the total distance to impact, speed at impact, perception/reaction time and deceleration factor, **mi/hr**.

\[ S_o = 21.96 fT + \sqrt{(21.96 fT)^2 + Sf^2 - 30fd} \]

\( S_o = \text{Speed original, mi/hr} \)
\( f = \text{Deceleration factor, decimal} \)
\( d = \text{Total distance to Impact, ft} \)
\( T = \text{Perception/Reaction time, sec} \)
**Maximum Distance**

5. Distance required to perceive/react and stop to avoid a hazard from a known velocity, \( \text{ft} \).

\[
d = \frac{V^2}{2fg} + VT
\]

\( V = \text{Velocity, ft/sec} \)
\( f = \text{Deceleration factor, decimal} \)
\( T = \text{Perception/Reaction time, sec} \)
\( g = \text{Gravitational constant, 32.2 ft/sec}^2 \)

6. Total distance required including perception/reaction time to decelerate from one velocity to another, \( \text{ft} \).

\[
d = VoT + \left( \frac{Vo^2}{2f} - \frac{Vf^2}{2f} \right)
\]

\( f = \text{Deceleration factor, decimal} \)
\( Vo = \text{Velocity original, ft/sec} \)
\( Vf = \text{Velocity final, ft/sec} \)
\( g = \text{Gravitational constant, 32.2 ft/sec}^2 \)
\( T = \text{Perception/Reaction time, sec} \)

**Collinear Impact**

* For equations 1 through 3, vehicles must depart after collision as one unit.

**Closing Velocity**

1. Closing velocity of a trailing vehicle on the lead vehicle in a collinear collision, \( \text{ft/sec} \).

\[V_C = \sqrt{2gE_D(W_T + W_L)/W_TW_L}\]

\( E_D = \text{Total combined crush energy for both vehicles, ft-lb} \)
\( W_T = \text{Weight, trailing vehicle, lb} \)
\( W_L = \text{Weight, lead vehicle, lb} \)
\( g = \text{Gravitational constant, 32.2 ft/sec}^2 \)
2. Pre-impact velocity of the closing vehicle (trailing vehicle) in a collinear collision, $\text{ft/sec}$.

$$ V_T = V_C W_L / (W_T + W_L) + V' $$

$V_T$ = Post-impact velocity of both vehicles as one unit, $\text{ft/sec}$

$V_C$ = Closing velocity of trailing vehicle on the lead vehicle, $\text{ft/sec}$ (Eq #1)

$W_T$ = Weight of trailing vehicle, lb

$W_L$ = Weight of lead vehicle, lb

$V' = $ Post-impact velocity of both vehicles as one unit, $\text{ft/sec}$

3. Velocity of the lead vehicle knowing the closing velocity and pre-impact velocity of the trailing vehicle, $\text{ft/sec}$.

$$ V_L = V_T - V_C $$

$V_L$ = Velocity of the lead vehicle knowing the closing velocity and pre-impact velocity of the trailing vehicle, $\text{ft/sec}$ (Eq #2)

$V_T$ = Pre-impact velocity of trailing vehicle, $\text{ft/sec}$ (Eq #2)

$V_C$ = Closing velocity of trailing vehicle on the lead vehicle, $\text{ft/sec}$ (Eq #1)

4. Closing velocity of two vehicles during a collinear impact, $\text{ft/sec}$.

$$ V_C = \sqrt{\frac{2E_d g}{W_1 + W_2}} \left( \frac{1}{W_1 W_2} \right) $$

$E_d$ = Total absorbed energy for damage from both vehicles, ft-lb

$W_1$ = Weight of vehicle #1, lb

$W_2$ = Weight of vehicle #2, lb

$e$ = Coefficient of restitution, decimal

$g$ = Gravitational constant, 32.2 ft/sec$^2$

Wells, Atkinson, Hennessy
5. Velocity for vehicle #1; Inline Collision; Vehicles traveling in same direction, \textbf{ft/sec}.

\[ V_1 = V_3 + \frac{W_2}{W_1}(V_4 - V_2) \]

\[ W_1 = \text{Weight, vehicle #1, lb} \]
\[ W_2 = \text{Weight, vehicle #2, lb} \]
\[ V_2 = \text{Pre-impact velocity veh #2, ft/sec} \]
\[ V_3 = \text{Post-impact velocity veh #1, ft/sec} \]
\[ V_4 = \text{Post-impact velocity veh #2, ft/sec} \]

6. Velocity veh #1 Inline Collision; Elastic (minimal damage), \textbf{ft/sec}.

\[ V_1 = \frac{V_4(1 + W_2 / W_1) + V_2(1 - W_2 / W_1)}{2} \]

\[ W_1 = \text{Weight, vehicle #1, lb} \]
\[ W_2 = \text{Weight, vehicle #2, lb} \]
\[ V_2 = \text{Pre-impact velocity veh #2, ft/sec} \]
\[ V_4 = \text{Post-impact velocity veh #2, ft/sec} \]

7. Velocity veh #1; Inline Collision, utilizing a coefficient of restitution, \textbf{ft/sec}.

\[ V_1 = \frac{V_4(1 + W_2 / W_1) + V_2(e - W_2 / W_1)}{1 + e} \]

\[ e = \text{Coefficient of Restitution, decimal} \]
\[ W_1 = \text{Weight, vehicle #1, lb} \]
\[ W_2 = \text{Weight, vehicle #2, lb} \]
\[ V_2 = \text{Pre-impact velocity veh #2, ft/sec} \]
\[ V_4 = \text{Post-impact velocity veh #2, ft/sec} \]

8. Velocity for vehicle #3; Inline Collision; Vehicles traveling in same direction, \textbf{ft/sec}.

\[ V_3 = V_1 - \frac{W_2}{W_1}(V_4 - V_2) \]

\[ W_1 = \text{Weight, vehicle #1, lb} \]
\[ W_2 = \text{Weight, vehicle #2, lb} \]
\[ V_1 = \text{Pre-impact velocity veh #1, ft/sec} \]
\[ V_2 = \text{Pre-impact velocity veh #2, ft/sec} \]
\[ V_4 = \text{Post-impact velocity veh #2, ft/sec} \]
**Coefficient of Restitution**

9. Coefficient of restitution, **decimal.** (Collinear impacts)

\[ e = \frac{(V_3 - V_4)}{(V_1 - V_2)} \]

- \( V_1 \) = Pre-impact velocity veh #1, ft/sec
- \( V_2 \) = Pre-impact velocity veh #2, ft/sec
- \( V_3 \) = Post-impact velocity veh #1, ft/sec
- \( V_4 \) = Post-impact velocity veh #2, ft/sec

For perfect elastic collision \( e = 1 \).
For inelastic collisions \( e < 1 \).
If vehicles lodge together after collision, \( V_4 = V_3 \), \( e = 0 \).

**Safe Following Distance**

10. Safe following distance between a lead and trailing vehicle prior to a collinear collision, **ft.**

\[ d_s = d + V\left(T_p + T_r + \frac{V}{2}(1/a_r - 1/a_L)\right) \]

- \( V \) = Initial velocity of vehicles, ft/sec
- \( d \) = Distance between vehicles at points of rest, ft
- \( a_L \) = Lead vehicle deceleration rate, ft/sec^2
- \( a_r \) = Trailing vehicle deceleration rate, ft/sec^2
- \( T_p \) = Trailing vehicle perception time, sec
- \( T_r \) = Trailing vehicle reaction time, sec
Frontal Sideswipe

11. Determine the pre-impact speed for vehicle #1 for in-line/sideswipe frontal collisions, \( \text{ft/sec} \).

\[
V_1 = \frac{m_2}{(m_1 + m_2)} \left(\frac{1}{m_2} m_1 V_3 - m_2 V_4\right) + \sqrt{\left(V_3 + V_4\right)^2 + \frac{m_1 + m_2}{m_2} bev_1^2 + \frac{m_1 + m_2}{m_1} bev_2^2}
\]

*Limpert*

\( bev_1 \) = Barrier equivalent velocity for vehicle #1, \( \text{ft/sec} \)  
(Eq #8 Crush Damage section)

\( bev_2 \) = Barrier equivalent velocity for vehicle #2, \( \text{ft/sec} \)  
(Eq #8 Crush Damage section)

\( m_1 \) = Mass of vehicle #1, \( \text{lb}-\text{sec}^2/\text{ft} \)

\( m_2 \) = Mass of vehicle #2, \( \text{lb}-\text{sec}^2/\text{ft} \)

\( V_3 \) = Post-impact velocity veh #1, \( \text{ft/sec} \)

\( V_4 \) = Post-impact velocity veh #2, \( \text{ft/sec} \)

12. Determine the pre-impact speed for vehicle #2 for in-line/sideswipe frontal collisions, \( \text{ft/sec} \).

\[
V_2 = V_4 - \left(\frac{m_1}{m_2}\right) V_3 + \left(\frac{m_1}{m_2}\right) V_1
\]

*Limpert*  

\( V_1 \) = Pre-impact velocity veh #1, \( \text{ft/sec} \)  
(Eq #11)

\( m_1 \) = Mass of vehicle #1, \( \text{lb}-\text{sec}^2/\text{ft} \)

\( m_2 \) = Mass of vehicle #2, \( \text{lb}-\text{sec}^2/\text{ft} \)

\( V_3 \) = Post-impact velocity veh #1, \( \text{ft/sec} \)

\( V_4 \) = Post-impact velocity veh #2, \( \text{ft/sec} \)
Rear end Sideswipe

13. Determine the pre-impact speed for vehicle #1 for in-line/sideswipe rear end collisions, ft/sec.

\[
V_1 = \frac{m_2}{(m_1 + m_2)} \left[ 1 + \frac{m_1 V_3 + m_2 V_4}{(m_1 + m_2) V_3} \right] + \sqrt{\left( \frac{m_1 + m_2}{m_2} \right) V_1^2 + \left( \frac{m_1 + m_2}{m_1} \right) V_2^2}
\]

\textbf{Limpert}

bev_1 = \text{Barrier equivalent velocity for vehicle #1, ft/sec (Eq #8 Crush Damage section)}

bev_2 = \text{Barrier equivalent velocity for vehicle #2, ft/sec (Eq #8 Crush Damage section)}

m_1 = \text{Mass of vehicle #1, lb-sec}^2/\text{ft}

m_2 = \text{Mass of vehicle #2, lb-sec}^2/\text{ft}

V_3 = \text{Post-impact velocity veh #1, ft/sec}

V_4 = \text{Post-impact velocity veh #2, ft/sec}

14. Determine the pre-impact speed for vehicle #2 for in-line/sideswipe rear end collisions, ft/sec.

\[
V_2 = V_4 + \left( \frac{m_1}{m_2} \right) V_3 - \left( \frac{m_1}{m_2} \right) V_1
\]

\textbf{Limpert}

m_1 = \text{Mass of vehicle #1, lb-sec}^2/\text{ft}

m_2 = \text{Mass of vehicle #2, lb-sec}^2/\text{ft}

V_1 = \text{Pre-impact velocity veh #1, ft/sec (Eq #13)}

V_3 = \text{Post-impact velocity veh #1, ft/sec}

V_4 = \text{Post-impact velocity veh #2, ft/sec}

Post Impact Speed

15. Post impact speed of Veh #2 during a collinear collision with vehicle #2 stationary prior to impact, ft/sec.

\[
V_4 = \left[ \frac{W_1}{(W_1 + W_2)} \right] (1-e) V_1
\]

\textbf{Wells, Atkinson, Hennessy}

W_1 = \text{Weight of vehicle #1, lb}

W_2 = \text{Weight of vehicle #2, lb}

e = \text{Coefficient of restitution, decimal}

V_1 = \text{Pre-impact speed of Veh #1, ft/sec}
16. Post impact speed of Veh #1 during a collinear collision with vehicle #2 stationary prior to impact, \( \text{ft/sec} \).

\[
V_3 = \left(\frac{W_1 + eW_2}{W_1 + W_2}\right)V_1
\]

\( W_1 = \text{Weight of vehicle #1, lb} \)

\( W_2 = \text{Weight of vehicle #2, lb} \)

\( e = \text{Coefficient of restitution, decimal} \)

\( V_1 = \text{Pre-impact speed of Veh #1, ft/sec} \)

**Delta V**

17. Delta V for the bullet vehicle during a collinear impact, \( \text{ft/sec} \).

\[
\Delta V_B = \frac{2E_DgW_2(1-e)^2}{\sqrt{W_1(W_1 + W_2)(1-e^2)}}
\]

\( E_D = \text{Total absorbed energy for damage from both vehicles, ft-lb} \)

\( W_1 = \text{Weight of vehicle #1, lb} \)

\( W_2 = \text{Weight of vehicle #2, lb} \)

\( e = \text{Coefficient of restitution, decimal} \)

\( g = \text{Gravitational constant, 32.2 ft/sec}^2 \)

18. Delta V for the target vehicle during a collinear impact, \( \text{ft/sec} \).

\[
\Delta V_T = \frac{2E_DgW_1(1-e)^2}{\sqrt{W_2(W_1 + W_2)(1-e^2)}}
\]

\( E_D = \text{Total absorbed energy for damage from both vehicles, ft-lb} \)

\( W_1 = \text{Weight of vehicle #1, lb} \)

\( W_2 = \text{Weight of vehicle #2, lb} \)

\( e = \text{Coefficient of restitution, decimal} \)

\( g = \text{Gravitational constant, 32.2 ft/sec}^2 \)
**Damage Crush**

The following variables are used in equations 1 through 10 of this section:

- \( A \) = Stiffness coefficient, lb/in
- \( B \) = Stiffness coefficient, lb/in\(^2\)
- \( G \) = Stiffness coefficient, lb
- \( g \) = Acceleration of gravity, 32.2 ft/sec\(^2\) (386.4 in/sec\(^2\))
- \( b_o \) = Intercept (maximum barrier velocity w/o permanent damage), in/sec
  
  (4.398 to 10.262 ft/sec or 52.776 to 123.144 in/sec)
- \( b_i \) = Slope of the speed versus crush relation, 1/sec (change in impact speed to the change in crush)
- \( E \) = Energy dissipated due to crush, in-lb
- \( L_C \) = Width of crush region (crash vehicle), in
- \( L_T \) = Width of crush region (test vehicle), in
- \( W_T \) = Total static weight of the test vehicle, lb
- \( \sigma \) = Angle of attack at impact, deg (angle between the PDOF (\( \rho \)) and the damaged side axis)
  
  Do not exceed 45 degrees.
- \( V_{imp} \) = Impact velocity of test vehicle, ft/sec
- \( C_{rave} \) = Average crush depth of test vehicle, in
- \( C_1 \) through \( C_6 \) = Crush measurements, in

**Centroid of Damage**

1. Centroid of Damage measured from the center of the damage width along the \( x \)-axis direction (depth), in.

\[
x = \frac{C_1^2 + 2C_2^2 + 2C_3^2 + 2C_4^2 + C_6^2 + C_1C_2 + C_2C_3 + C_3C_4 + C_4C_5 + C_5C_6}{3(C_1 + 2C_2 + 2C_3 + 2C_4 + 2C_5 + C_6)}
\]
2. Centroid of Damage measured from the center of the damage width along the \( y \)-axis direction (width), \text{in}.

\[
y = \left( \frac{L}{30} \right) \left( \frac{-13C_1 - 18C_2 - 6C_3 + 6C_4 + 18C_5 + 13C_6}{C_1 + 2C_2 + 2C_3 + 2C_4 + 2C_5 + C_6} \right)
\]

The following variables (\( g, b, \) and \( V \text{imp} \)) are converted to \text{in/sec} prior to their entry into the following equations. This is done by multiplying the variables (ft/sec) by 12. Use the crush data available by NHTSA.

3. Slope of the speed versus crush relation, \( \text{1/sec} \).

\[
Campbell \quad b_1 = \frac{V \text{imp} - b_o}{Cr_{ave}}
\]

4. Maximum force per inch of damage width without permanent damage, \( \text{lb/in} \).

\[
Campbell \quad A = \frac{W_T \cdot b_o b_1}{gL_T}
\]

5. Crush resistance per inch of damage width, \( \text{lb/in}^2 \).

\[
Campbell \quad B = \frac{W_T \cdot b_1^2}{gL_T}
\]

6. Energy dissipated without permanent damage, \( \text{lb} \).

\[
Campbell \quad G = A^2 / (2B)
\]
Table 1: Stiffness Values; Average

<table>
<thead>
<tr>
<th>Vehicle Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Passenger Cars</strong></td>
<td></td>
</tr>
<tr>
<td>Frontal Crash</td>
<td>A = 325 ± 20 %</td>
</tr>
<tr>
<td></td>
<td>B = 43 ± 20 %</td>
</tr>
<tr>
<td>Rear-end Crash</td>
<td>A = 364 ± 20 %</td>
</tr>
<tr>
<td></td>
<td>B = 48 ± 20 %</td>
</tr>
<tr>
<td>Side Crash</td>
<td>A = 142 ± 35 %</td>
</tr>
<tr>
<td></td>
<td>B = 52 ± 35 %</td>
</tr>
<tr>
<td><strong>Pickup Trucks</strong></td>
<td></td>
</tr>
<tr>
<td>Frontal Crash</td>
<td>A = 456 ± 10 %</td>
</tr>
<tr>
<td></td>
<td>B = 90 ± 25 %</td>
</tr>
<tr>
<td>Rear-end Crash</td>
<td>A = 350 ± 20 %</td>
</tr>
<tr>
<td></td>
<td>B = 25 ± 20 %</td>
</tr>
<tr>
<td>Side Crash</td>
<td>A = 60 ± 20 %</td>
</tr>
<tr>
<td></td>
<td>B = 45 ± 20 %</td>
</tr>
<tr>
<td><strong>Vans</strong></td>
<td></td>
</tr>
<tr>
<td>Frontal Crash</td>
<td>A = 380 ± 20 %</td>
</tr>
<tr>
<td></td>
<td>B = 125 ± 20 %</td>
</tr>
<tr>
<td>Rear-end Crash</td>
<td>A = 300 ± 20 %</td>
</tr>
<tr>
<td></td>
<td>B = 55 ± 20 %</td>
</tr>
</tbody>
</table>

**Damage Profile**

Place the above values for \((A, B, G)\) into one of the following Damage Profile Equations:

7. Two Point Damage Profile (crush energy), **in-lb**:

\[
E = (1 + \tan^2 \sigma) \left( L_c \left[ \frac{A}{2} \left( C_1 + C_2 \right) + \frac{B}{6} \left( C_1^2 + C_1 C_2 + C_2^2 \right) + G \right] \right)
\]

8. Four Point Damage Profile (crush energy), **in-lb**:

\[
E = \left(1 + \tan^2 \sigma\right) \left( \frac{L_c}{3} \left[ \frac{A}{2} \left( C_1 + 2C_2 + 2C_3 + C_4 \right) + \frac{B}{6} \left( \frac{C_1^2 + 2C_2^2 + 2C_3^2 + C_4^2}{C_1 C_2 + C_2 C_3 + C_3 C_4} \right) + 3G \right] \right)
\]
9. Six Point Damage Profile (crush energy), **in-lb**:

\[
E = \left(1 + \tan^2 \sigma\right) \left(\frac{L_c}{5}\right) \left(\frac{A}{2}\right) \left(\frac{C_1 + 2C_2 + 2C_3}{2C_4 + 2C_5 + C_6}\right) + \left(\frac{B}{6}\right) \left(\frac{C_1^2 + 2C_2^2 + 2C_3^2 + 2C_4^2 + 2C_5^2 + C_6^2}{C_1C_2 + 2C_2C_3 + C_3C_4 + C_4C_5 + C_5C_6}\right) + 5G
\]

**Average Crush Depth**

10. \[C_{ave} = \frac{C_1/2 + C_2 + C_3 + C_4 + C_5 + C_6/2}{5}\] Average crush depth for a six point damage profile, **in**.

**Barrier Equivalent Velocity**

The barrier equivalent velocity (bev) ft/sec can then be calculated from the energy \(E\) in-lb produced from the above Damage Profile Equations.

First, rewrite the variable \(E\) from in-lb to ft-lb by division of 12. Then place the variable \(E\) ft-lb into the following equation:

11. Barrier equivalent velocity utilizing energy, **ft/sec**.

\[
bev = \sqrt{\frac{2E\gamma}{m}}
\]

\(E\) = Collision energy dissipated due to crush, ft-lb (Eq #8 thru 10)
\(m\) = Mass of vehicle, lb-sec²/ft
\(\gamma\) = Effective mass coefficient at the center of gravity, decimal (Eq #24)

12. Barrier equivalent velocity utilizing energy, **ft/sec**.

\[
bev = \sqrt{\frac{2gE}{W}}
\]

\(E\) = Collision energy dissipated due to crush, ft-lb (Eq #8 thru 10)
\(W\) = Total static weight, lb
\(g\) = Gravitational constant, 32.2 ft/sec²
13. Delta V due to crush for vehicle #1, \textbf{ft/sec}.

\[ \Delta V_1 = \sqrt{2 \left( E_1 + E_2 \right) / \left( m_1 \left( 1 + m_1 / m_2 \right) \right)} \]

\( E_1 \) = Collision energy dissipated due to crush, vehicle #1, ft-lb (Eq #8 thru 10)

\( E_2 \) = Collision energy dissipated due to crush, vehicle #2, ft-lb (Eq #8 thru 10)

\( m_1 \) = Mass of vehicle #1, lb-sec\(^2\)/ft

\( m_2 \) = Mass of vehicle #2, lb-sec\(^2\)/ft

14. Delta V due to crush for vehicle #2, \textbf{ft/sec}.

\[ \Delta V_2 = \sqrt{2 \left( E_1 + E_2 \right) / \left( m_2 \left( 1 + m_2 / m_1 \right) \right)} \]

\( E_1 \) = Collision energy dissipated due to crush, vehicle #1, ft-lb

\( E_2 \) = Collision energy dissipated due to crush, vehicle #2, ft-lb

\( m_1 \) = Mass of vehicle #1, lb-sec\(^2\)/ft

\( m_2 \) = Mass of vehicle #2, lb-sec\(^2\)/ft

15. Delta V for either vehicle #1 or 2, \textbf{ft/sec}.

\[ \Delta V = P / m \]

\( P \) = Impulse, lb-sec (Eq #27)

\( m \) = Mass, lb-sec\(^2\)/ft

16. Longitudinal component of a Delta V, \textbf{ft/sec}.

\[ \Delta V_{\text{long}} = \Delta V \cos(\rho - 180) \]

\( \Delta V \) = Delta V. Magnitude of the velocity change for the center of gravity, ft/sec (Eq #13, 14, 15)

\( \rho \) = Principal direction of force, deg

(Eq #14, 15 Momentum section)
17. Lateral component of a Delta V, \( \text{ft/sec} \).

\[
\Delta V_{\text{lat}} = \Delta V \sin(\rho-180) \\
\Delta V = \text{Delta V. Magnitude of the velocity change for the center of gravity, ft/sec (Eq #13, 14, 15)} \\
\rho = \text{Principal direction of force, deg} \\
\text{(Momentum section)}
\]

**Angular Velocity**

18. Angular velocity of a post-impact rotation, \( \text{rad/sec} \).

\[
\omega = 0.132 \sqrt{\frac{W f \alpha}{I}} \\
f = \text{Rotational friction coefficient, decimal} \\
\ell = \text{Wheelbase, ft} \\
\alpha = \text{Post-impact angle after rotation, deg} \\
W = \text{Total static weight, lb} \\
I = \text{Mass moment of inertia, ft-lb-sec}^2 \\
\text{(Eq #5, 6 Mass section)}
\]

**Limpert**

19. Angular velocity of a post-impact rotation, \( \text{rad/sec} \).

\[
\omega = \left( \frac{\pi \text{rpm}}{30} \right) \\
\pi = \text{Pi, 3.141592654} \\
\text{rpm} = \text{Revolutions per minute}
\]

20. Delta angular velocity, \( \text{rad/sec} \).

\[
\Delta \omega = \left( \frac{F_a P}{k^2 m} \right) \\
F_a = \text{Moment arm of force, in (Eq #25)} \\
P = \text{Impulse, lb-sec (Eq #27)} \\
k^2 = \text{Radius of gyration, in}^2 \text{ (Eq #29 thru 31)} \\
m = \text{Mass, lb-sec}^2/\text{in} \text{ (Eq #2 Mass section)}
\]

SAE #870045
Rotation Time

21. Time of rotation after impact, **sec**.

\[ T = \frac{2\,\theta}{\omega} \]

\(\theta = \text{Total degree of rotation, rad}\)

\(\omega = \text{Angular velocity, rad/sec (Eq #18, 19)}\)

Energy Correction Factor

22. Energy correction factor (mag factor), **decimal**. Do not utilize with Eq #8 thru 10 of this chapter. This equation is already incorporated into their solution.

\[ M_F = 1 + \tan^2 \sigma \]

\(\sigma = \text{Angle of attack at impact, deg}\)

(angle between the PDOF (\(\rho\)) and the damaged side axis) Do not exceed 45 degrees.

Effective Mass Coefficient

23. Effective mass coefficient at the center of gravity (gamma), **decimal**.

\[ \gamma = \frac{k^2}{\left(k^2 + F_a^2\right)} \]

\(k^2 = \text{Radius of gyration, in}^2\) (Eq #29 thru 31)

\(F_a = \text{Moment arm of force, in}\) (Eq #25)

Collision Force

24. Collision force at the centroid of damage utilizing stiffness coefficients, **lb**.

\[ F = \left(A + BC_{ave}\right)\frac{L}{\cos \sigma} \]

\(A = \text{Constant (stiffness coefficient), lb/in (Eq #4)}\)

\(B = \text{Constant (stiffness coefficient), lb/in}^2\) (Eq #5)

\(L = \text{Width of crush region, in}\)

\(\sigma = \text{Angle of attack at impact, deg (angle between the PDOF (\(\rho\)) and the damaged side axis) Do not exceed 45 degrees.}\)

\(C_{ave} = \text{Weighted average crush of vehicle, in}\)

(Equally spaced points)
**Moment Arm of Force**

25. Determine the offset (moment arm of force) from the center of gravity to the line of force applied through the centroid of damage, knowing the center of gravity coordinates to the centroid of damage, **in**. Positive value for clockwise rotation to the vehicle.

\[ F_a = y_c \cos \rho - x_c \sin \rho \]

\( x_c \) = Distance from center of gravity to centroid of damage in the \( x \) direction, **in**  
(Damage Centroid section)

\( y_c \) = Distance from center of gravity to centroid of damage in the \( y \) direction, **in**  
(Damage Centroid section)

\( \rho \) = Principal direction of force, **deg**  
(Eq #14, 15 Momentum section)

**Effective Force**

26. Effective force which is parallel to the direction of travel for vehicle #1, **lb**.

\[ F_e = F \cos \rho_1 \]

\( F \) = Force, **lb**  
(Eq #24)

\( \rho_1 \) = Principal direction of force, **deg**  
(Momentum section)

**Impulse**

27. Determine a common impulse for both vehicles, **lb-sec**.

\[ P = \sqrt{2E_T \gamma_1 m_1 \gamma_2 m_2 / ((\gamma_1 m_1) + (\gamma_2 m_2))} \]

\( \gamma_1 \) = Effective mass coefficient for vehicle #1, decimal  
(Eq #23)

\( \gamma_2 \) = Effective mass coefficient for vehicle #2, decimal  
(Eq #23)

\( m_1 \) = Mass of vehicle #1, **lb-sec**^2/ft  
\( m_2 \) = Mass of vehicle #2, **lb-sec**^2/ft

\( E_T \) = Total combined damage energy of both vehicles due to crush, **ft-lb**  
(Eq #7, 8, 9)
Common Closing Velocity

28. Common closing velocity (relative approach velocity) for the line of action of the collision forces, $\text{ft/sec}$.

$$V_C = \frac{\Delta V_1}{\gamma_1} + \frac{\Delta V_2}{\gamma_2}$$

$\Delta V_1 = \text{Delta } V$. Magnitude of the velocity change for the center of gravity for vehicle #1, $\text{ft/sec}$ (Eq # 13 , 15)

$\Delta V_2 = \text{Delta } V$. Magnitude of the velocity change for the center of gravity for vehicle #2, $\text{ft/sec}$ (Eq #14, 15)

$\gamma_1 = \text{Effective mass coefficient for vehicle #1, decimal}$ (Eq #23)

$\gamma_2 = \text{Effective mass coefficient for vehicle #2, decimal}$ (Eq #23)

Yaw Radius of Gyration

29. Yaw radius of gyration, $\text{in}^2$. (Vehicles less than 6000 lbs)

$$k^2 = \left(0.298 \pm 0.030\right)L^2$$

$L = \text{Overall length of vehicle, in}$

Garrott

30. Yaw radius of gyration, $\text{in}^2$. (Vehicles less than 6000 lbs)

$$k^2 = \left(1.023 \pm 0.085\right)\sqrt{x_Fx_R}$$

$x_F = \text{Longitudinal center of mass measured from the front axle, in}$

Garrott

$x_R = \text{Longitudinal center of mass measured from the rear axle, in}$

SAE 930897

31. Yaw radius of gyration, $\text{in}^2$.

$$k^2 = (0.37\ell + 18.1)^2$$

$\ell = \text{Wheelbase, in}$
**Delta Time**

32. Change in time during a collision (duration of impact; typically 0.1 - 0.2 seconds), \textbf{sec.}

\[ \Delta T = \frac{P}{F} \]

*P* = Impulse, lb-sec (Eq #27)

*F* = Collision force at the centroid of damage, lb (Eq #24)

**Coefficient of Restitution**

33. Coefficient of restitution, \textbf{decimal}. (Collinear impacts)

\[ e = \frac{(V_3 - V_4)}{(V_1 - V_2)} \]

For perfect elastic collision \( e = 1 \).

For inelastic collisions \( e < 1 \).

If vehicles lodge together after collision, \( V_4 = V_3, \ e = 0 \).

\( V_1 \) = Pre-impact velocity veh #1, ft/sec

\( V_2 \) = Pre-impact velocity veh #2, ft/sec

\( V_3 \) = Post-impact velocity veh #1, ft/sec

\( V_4 \) = Post-impact velocity veh #2, ft/sec

**Damage (Rigid Pole Impact)**

The utility pole must act as a rigid barrier by stopping or significantly slowing the colliding vehicle.

**Equivalent Barrier Speed**

1. Equivalent barrier speed due to rigid pole frontal impact knowing maximum crush depth, \textbf{mph}. Utilize variables from table below for the intercept and slope

\[ ebs = b_o + b_1 Cr \]

*Cr* = Maximum crush depth, in

\( b_o \) = Intercept (maximum barrier velocity w/o permanent damage), mph

\( b_1 \) = Slope of the speed versus crush relation, mph/in (change in impact...
speed to the change in crush)

<table>
<thead>
<tr>
<th>Vehicle Weight</th>
<th>( h_1 )</th>
<th>( b_o )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1950 – 2450</td>
<td>0.641</td>
<td>3.04</td>
</tr>
<tr>
<td>2450 – 2950</td>
<td>0.648</td>
<td>2.46</td>
</tr>
<tr>
<td>2950 – 3450</td>
<td>0.600</td>
<td>4.04</td>
</tr>
<tr>
<td>3450 – 3950</td>
<td>0.516</td>
<td>4.84</td>
</tr>
<tr>
<td>3950 – 4450</td>
<td>0.467</td>
<td>4.33</td>
</tr>
</tbody>
</table>

2. Equivalent barrier speed due to rigid pole frontal impact, **mi/hr**. Utilize for 15 inches of crush or less.

\[
ebs = \frac{Cr\sqrt{1.27 - 0.0002W}}{W} \quad Cr = \text{Maximum crush depth, in} \quad W = \text{Total static weight, lb}
\]

*Morgan & Ivey*

3. Barrier equivalent velocity due to rigid pole frontal impact, **ft/sec**. Utilize for 1.25 feet of crush or less.

\[
bev = Cr\sqrt{(395 - 0.062W)(1 + \Delta E)} \quad Cr = \text{Maximum crush depth, ft} \quad W = \text{Total static weight, lb} \quad \Delta E = \text{Energy absorbed to the vehicle during crush, decimal}
\]

*Morgan & Ivey*  
*SAE 870607*  
The following values recommended:  
+0.25 for 3-40 class utility pole  
0.0 for 4-40 class utility pole  
-0.25 for 5-40 class utility pole
4. Equivalent barrier speed due to rigid pole frontal impact, \textbf{mi/hr}. Utilize for 15 inches of crush or less. Autos less than 2450 lb

\[ ebs = 0.641Cr + 3.04 \quad Cr = \text{Maximum crush depth, in} \]

5. Equivalent barrier speed due to rigid pole frontal impact, \textbf{mi/hr}. Utilize for 15 inches of crush or less. Autos 2450 to 2950 lb

\[ ebs = 0.648Cr + 2.46 \quad Cr = \text{Maximum crush depth, in} \]

6. Equivalent barrier speed due to rigid pole frontal impact, \textbf{mi/hr}. Utilize for 15 inches of crush or less. Autos 2950 to 3450 lb

\[ ebs = 0.600Cr + 4.04 \quad Cr = \text{Maximum crush depth, in} \]

7. Equivalent barrier speed due to rigid pole frontal impact, \textbf{mi/hr}. Utilize when crush exceeds 15 inches.

\[ ebs = (0.964 - 0.0000351W)Cr + b_o \quad b_o = \text{Intercept (maximum barrier speed w/o permanent damage), mi/hr} \]

\textit{Nystrom & Kost}

\textit{SAE 920605}

\[ Cr = \text{Maximum crush depth, in} \]

\[ W = \text{Total static weight, lb} \]
8. Equivalent barrier speed due to rigid pole frontal or side impact, **mi/hr**.

\[ ebs = 1.02Cr \]  
\[ Cr = \text{Maximum crush depth, in} \]

9. The following crush formulas are based on "Rule of Thumb";
1 inch maximum crush = 1 mi/hr. The resultant is in **mi/hr**. Utilize with rigid pole frontal impacts.

\[ Craig \]
\[ Cr = \text{Maximum crush depth, in} \]

\[
\begin{align*}
    ebs &= Cr \\
    ebs &= Cr + 1 \\
    ebs &= Cr + 2 \\
    ebs &= Cr + 3 \\
    ebs &= Cr + 4 \\
    ebs &= Cr + 5
\end{align*}
\]

15 to 20 in. maximum crush
21 to 25 in. maximum crush
26 to 30 in. maximum crush
31 to 35 in. maximum crush
36 to 40 in. maximum crush
41 to 45 in. maximum crush

10. Equivalent barrier speed due to rigid pole frontal impact, **mi/hr**. Utilize for 12 inches of crush or less. **Compact/Sub Compact front wheel drive vehicles**.

\[ ebs = 0.47Cr + 4.0 \]  
\[ Cr = \text{Maximum crush depth, in} \]

\[ Craig \]

11. Equivalent barrier speed due to rigid pole frontal impact, **mi/hr**. Utilize for 12 inches of crush or more. **Compact/Sub Compact front wheel drive vehicles**.

\[ ebs = 1.3Cr - 6 \]  
\[ Cr = \text{Maximum crush depth, in} \]

\[ Craig \]
12. Equivalent barrier speed due to rigid pole frontal impact, \textbf{mi/hr}. Utilize for 18 inches of crush or less. \textbf{Large vehicles over 180 inches.}

\[ ebs = 0.54Cr + 4.0 \quad Cr = \text{Maximum crush depth, in} \]

\textit{Craig}

13. Equivalent barrier speed due to rigid pole frontal impact, \textbf{mi/hr}. Utilize for 18 inches of crush or more. \textbf{Large vehicles over 180 inches.}

\[ ebs = 1.18Cr - 7 \quad Cr = \text{Maximum crush depth, in} \]

\textit{Craig}

14. Equivalent barrier speed due to rigid pole frontal impact, \textbf{mi/hr}. Utilize for 12 inches of crush or less. \textbf{Mini vehicles.}

\[ ebs = 1.0Cr + 4 \quad Cr = \text{Maximum crush depth, in} \]

\textit{Craig}

15. Equivalent barrier speed due to rigid pole frontal impact, \textbf{mi/hr}. Utilize for 12 inches of crush or more. \textbf{Mini vehicles.}

\[ ebs = 1.0Cr \quad Cr = \text{Maximum crush depth, in} \]

\textit{Craig}

\textbf{Coefficient of Restitution}

16. Coefficient of restitution due to rigid barrier impact, \textbf{decimal}.

\[ e = \left(-V_3\right)/V_1 \]

\[ V_1 = \text{Pre-impact velocity, ft/sec} \]

\[ V_3 = \text{Post-impact velocity, ft/sec} \]

\text{(utilized as a negative value (-) due to rebound)}
**Utility Pole Fracture**

17. Energy required to completely fracture a utility pole, \textbf{ft-lb}. The pole must have a circumference of at least 26 inches. (Breakaway Fracture Energy)

\[ BFE = 0.014c^{4.38} \]

\[ c = \text{Circumference of pole, in} \]

**Damage (Miscellaneous)**

1. Equivalent barrier speed utilizing damage associated with a test vehicle of the same caliber as the crash vehicle, \textbf{mi/hr}.

\[ ebs = \left( \frac{S_{imp} - b_o}{Cr_{ave}} \right)Cr + b_o \]

\[ Cr = \text{Average crush of crash vehicle, in} \]

\[ Cr_{ave} = \text{Average crush of test vehicle, in} \]

\[ S_{imp} = \text{Impact speed of test vehicle, mi/hr} \]

\[ b_o = \text{Intercept, mi/hr. Maximum barrier impact speed without permanent damage. Threshold of 3 to 7 mi/hr} \]

2. \textbf{Small vehicle, frontal impact}. Equivalent barrier speed due to crush, \textbf{mi/hr}. Directional force nothing other than zero.

\[ ebs = -22.6 + 11\sqrt{Cr_{ave} + 4.18} \]

\[ ebs = \pm 0.5Cr_{ave} \]

\[ Limpert \]

3. \textbf{Midsize vehicle, frontal impact}. Equivalent barrier speed due to crush, \textbf{mi/hr}. Directional force nothing other than zero.

\[ ebs = -14.2 + 9.4\sqrt{Cr_{ave} + 2.62} \]

\[ ebs = \pm 0.5Cr_{ave} \]

\[ Limpert \]
4. **Large vehicle, frontal impact.** Equivalent barrier speed due to crush, \(\text{mi/hr}\). Directional force nothing other than zero.

\[
ebs = -11.6 + 8.4 \sqrt{Cr_{\text{ave}} + 1.92} \quad \text{Cr}_{\text{ave}} = \text{Average crush (} C_1 \text{ thru } C_6 \text{), in}
\]

\[
ebs = \pm 0.5 Cr_{\text{ave}}
\]

*Limpert*

5. **All vehicles, rear impact.** Equivalent barrier speed due to crush, \(\text{mi/hr}\). Directional force nothing other than zero.

\[
ebs = 1.133 Cr_{\text{ave}} + 2.67 \quad \text{Cr}_{\text{ave}} = \text{Average crush (} C_1 \text{ thru } C_6 \text{), in}
\]

\[
ebs = \pm 0.5 Cr_{\text{ave}}
\]

*Limpert*

6. **All vehicles, side impact.** Equivalent barrier speed due to crush, \(\text{mi/hr}\). Directional force nothing other than zero.

\[
ebs = 551 - 46.8 \sqrt{139 - Cr_{\text{ave}}} \quad \text{Cr}_{\text{ave}} = \text{Average crush (} C_1 \text{ thru } C_6 \text{), in}
\]

\[
ebs = \pm 0.5 Cr_{\text{ave}}
\]

*Limpert*

7. **Rear end collision.** Equivalent barrier speed for a trailing vehicle from measured crush to the rear of the lead vehicle in a rear end collision, \(\text{mi/hr}\). *(Average for all passenger vehicles)*

\[
ebs = 1.4 Cr_{\text{ave}} \quad \text{Cr}_{\text{ave}} = \text{Average crush to front of trailing vehicle, in cm}
\]
8. **Frontal to side collision.** Equivalent barrier speed of a striking vehicle from combined crush of both vehicles, \( \text{mi/hr} \).

\[
ebs = 1.5Cr_{ave}
\]

\( Cr_{ave} \) = Combined average crush to both vehicles, in \( \text{cm} \)

9. The following is a list of Calspan frontal impact equations for 1974 and prior vehicles. Equivalent barrier speed due to crush, \( \text{mi/hr} \). Directional force nothing other than zero.

9a. **Full size US vehicles.**

\[
ebs = 1.0 + 1.25Cr_{ave}
\]

\( Cr_{ave} \) = Average crush (\( C_1 \) thru \( C_6 \)), in

9b. **Compact & midsize vehicles.**

\[
ebs = -0.3 + 1.8Cr_{ave}
\]

\( Cr_{ave} \) = Average crush (\( C_1 \) thru \( C_6 \)), in

9c. **Foreign front engine, rear drive.**

\[
ebs = 3.5 + 1.39Cr_{ave}
\]

\( Cr_{ave} \) = Average crush (\( C_1 \) thru \( C_6 \)), in

9d. **Foreign front wheel drive.**

\[
ebs = 6.7 + 1.36Cr_{ave}
\]

\( Cr_{ave} \) = Average crush (\( C_1 \) thru \( C_6 \)), in

9e. **Foreign rear engine.**

\[
ebs = -3.6 + 1.72Cr_{ave}
\]

\( Cr_{ave} \) = Average crush (\( C_1 \) thru \( C_6 \)), in

9f. **All vehicles.**

\[
ebs = 7 + 1.5Cr_{ave}
\]

\( Cr_{ave} \) = Average crush (\( C_1 \) thru \( C_6 \)), in
Delta V

Momentum

1. \( \Delta V_1 = \sqrt{V_1^2 + V_3^2} - 2V_1V_3\cos\theta \)  

Delta V for vehicle #1 at impact, \textbf{ft/sec}.

\[ V_1 = \text{Pre-impact velocity veh #1, ft/sec} \]  
\[ (\text{Eq #2a Momentum section}) \]

\[ V_3 = \text{Post-impact velocity veh #1, ft/sec} \]  
\[ (\text{Eq #2c Momentum section}) \]

\[ \theta = \text{Departure angle vehicle #1, deg} \]  
\[ (\text{Eq #22 Momentum section}) \]

2. \( \Delta V_2 = \sqrt{V_2^2 + V_4^2} - 2V_2V_4\cos\beta \)  

Delta V for vehicle #2 at impact, \textbf{ft/sec}.

\[ V_2 = \text{Pre-impact velocity veh #2, ft/sec} \]  
\[ (\text{Eq #2b Momentum section}) \]

\[ V_4 = \text{Post-impact velocity veh #2, ft/sec} \]  
\[ (\text{Eq #2d Momentum section}) \]

\[ \beta = \text{Subtraction of Veh #2’s departure angle from its angle of approach } (\psi - \phi), \text{ deg} \]

Crush

3. \( \Delta V_1 = \sqrt{2(E_1 + E_2) / (m_1(1 + m_1 / m_2))} \)  

Delta V due to crush for vehicle #1, \textbf{ft/sec}.

\[ E_1 = \text{Collision energy dissipated due to crush, vehicle #1, ft-lb} \]  
\[ (\text{Eq #5, 6, 7 Damage Crush section}) \]

\[ E_2 = \text{Collision energy dissipated due to crush, vehicle #2, ft-lb} \]  
\[ (\text{Eq #5, 6, 7 Damage Crush section}) \]

\[ m_1 = \text{Mass of vehicle #1, lb-sec}^2 /\text{ft} \]  
\[ m_2 = \text{Mass of vehicle #2, lb-sec}^2 /\text{ft} \]
4. \[ \Delta V_2 = \sqrt{2(E_1 + E_2)/(m_2(1 + m_2/m_1))} \] Delta V due to crush for vehicle #2, \textbf{ft/sec}.

\[ E_1 = \text{Collision energy dissipated due to crush, vehicle #1, ft-lb} \]
(Eq #5, 6, 7 Damage Crush section)

\[ E_2 = \text{Collision energy dissipated due to crush, vehicle #2, ft-lb} \]
(Eq #5, 6, 7 Damage Crush section)

\[ m_1 = \text{Mass of vehicle #1, lb-sec}^2/\text{ft} \]
\[ m_2 = \text{Mass of vehicle #2, lb-sec}^2/\text{ft} \]

5. \[ \Delta V = P/m \] Delta V for either vehicle #1 or 2, \textbf{ft/sec}.

\[ P = \text{Impulse, lb-sec} \]
(Eq #23 Damage Crush section)

\[ m = \text{Mass, lb-sec}^2/\text{ft} \]

\textit{Longitudinal Component}

6. \[ \Delta V_{\text{long}} = \Delta V \cos(\rho-180) \] Longitudinal component of a Delta V, \textbf{ft/sec}.

\[ \Delta V = \text{Delta V. Magnitude of the velocity change for the center of gravity, ft/sec} \] (Eq #1 thru 5)

\[ \rho = \text{Principal direction of force, deg} \]
(Eq #14, 15 Momentum section)

\textit{Lateral Component}

7. \[ \Delta V_{\text{lat}} = \Delta V \sin(\rho-180) \] Lateral component of a Delta V, \textbf{ft/sec}.

\[ \Delta V = \text{Delta V. Magnitude of the velocity change for the center of gravity, ft/sec} \] (Eq #1 thru 5)

\[ \rho = \text{Principal direction of force, deg} \]
(Eq #14, 15 Momentum section)
Common Closing Velocity

8. \[ V_c = \left( \Delta V_1 / \gamma_1 \right) + \left( \Delta V_2 / \gamma_2 \right) \]

Common closing velocity (relative approach velocity) for the line of action of the collision forces, \textbf{ft/sec}.

\[ \Delta V_1 = \text{Delta } V. \text{ Magnitude of the velocity change for the center of gravity for vehicle #1, ft/sec (Eq #1, 3)} \]

\[ \Delta V_2 = \text{Delta } V. \text{ Magnitude of the velocity change for the center of gravity for vehicle #2, ft/sec (Eq #2, 4)} \]

\[ \gamma_1 = \text{Effective mass coefficient for vehicle #1, decimal (Eq #18 Damage Crush section)} \]

\[ \gamma_2 = \text{Effective mass coefficient for vehicle #2, decimal (Eq #18 Damage Crush section)} \]

Distance

The length or numerical value of a straight line or curve.

1. Distance traveled over a unit of time, \textbf{ft}.

\[ d = VT \]

\[ V = \text{Constant velocity, ft/sec} \]

\[ T = \text{Time, sec} \]

2. Distance traveled over a unit of time, \textbf{ft}.

\[ d = 1.466ST \]

\[ S = \text{Constant speed, mi/hr} \]

\[ T = \text{Time, sec} \]
3. Distance traveled after accel/decelerating from or to a stop, ft.

\[ d = \frac{S^2}{30(\mu n + m)} \]

- \( S \) = Speed, mi/hr
- \( \mu \) = Level friction coefficient, decimal
- \( n \) = Grade, maximum 11.9%, decimal
- \( m \) = Grade, maximum 11.9%, decimal
- \( \mu \) = Level friction coefficient, decimal
- \( n \) = Braking efficiency, decimal (deceleration only)

4. Distance traveled after accel/decelerating from or to a stop, ft.

\[ d = 0.5aT^2 \]

- \( a \) = Accel / Decel rate, ft/sec²
- \( T \) = Time, sec

5. Distance traveled after accel/decelerating from or to a stop, ft.

\[ d = \frac{WS^2}{30F} \]

- \( S \) = Speed, mi/hr
- \( F \) = Force applied, lb
- \( W \) = Total static weight, lb

6. Distance traveled after accel/decelerating from or to a stop, ft.

\[ d = \frac{WV^2}{2gF} \]

- \( V \) = Velocity, ft/sec
- \( g \) = Gravitational constant, 32.2 ft/sec²
- \( F \) = Force applied, lb
- \( W \) = Total static weight, lb

7. Distance traveled after accel/decelerating from or to a stop over a unit of time, ft.

\[ d = 16.1(f \pm m)T^2 \]

- \( f \) = Accel / Decel factor, decimal
- \( m \) = Grade, maximum 11.9%, decimal
- \( \mu \) = Level friction coefficient, decimal
- \( n \) = Grade, maximum 11.9%, decimal
- \( \mu \) = Level friction coefficient, decimal
- \( n \) = Braking efficiency, decimal (deceleration only)
- \( T \) = Time, sec
8. Distance traveled after accel/decelerating from or to a stop over a unit of time, ft.

\[ d = \left( \frac{V}{2} \right) T \]

\( V = \) Velocity, ft/sec
\( T = \) Time, sec

9. Distance traveled after accel/decelerating to or from a stop, ft.

\[ d = \frac{V^2}{2(\pm f \pm m)} g \]

\( V = \) Velocity, ft/sec
\( f = \) Accel / Decel factor, decimal
\( m = \) Grade, maximum 11.9%, decimal
\( (\pm f) \) for incline, \((\pm m)\) for decline
\( g = \) Gravitational constant, 32.2 ft/sec^2

10. Distance traveled after accel/decelerating from or to a stop, ft.

\[ d = \frac{V^2}{2a} \]

\( V = \) Velocity, ft/sec
\( a = \) Accel / Decel rate, ft/sec^2

11. Distance traveled after accel/decelerating from an initial velocity over a unit of time, ft.

\[ d = V_0 T + 0.5aT^2 \]

\( V_0 = \) Velocity initial, ft/sec
\( T = \) Time, sec
\( a = \) Accel / Decel rate, ft/sec^2
\((\text{negative value} \ (-) \ \text{for deceleration})\)
12. Distance traveled after accel/decelerating from an initial velocity over a unit of time, ft.

\[ d = VoT + 0.5fgT^2 \]

- \( d \) = Distance traveled, ft
- \( Vo \) = Velocity initial, ft/sec
- \( T \) = Time, sec
- \( f \) = Accel / Decel factor, decimal
  (negative value (-) for deceleration)
- \( g \) = Gravitational constant, 32.2 ft/sec\(^2\)

13. Distance traveled after accel/decelerating from an initial speed over a unit of time, ft.

\[ d = 1.466SoT + 0.5aT^2 \]

- \( d \) = Distance traveled, ft
- \( So \) = Speed initial, mi/hr
- \( T \) = Time, sec
- \( a \) = Accel / Decel rate, ft/sec\(^2\)
  (negative value (-) for deceleration)

14. Distance traveled after accel/decelerating from an initial speed over a unit of time, ft.

\[ d = 1.466SoT + 0.5fgT^2 \]

- \( d \) = Distance traveled, ft
- \( So \) = Speed initial, mi/hr
- \( T \) = Time, sec
- \( f \) = Accel / Decel factor, decimal
  (negative value (-) for deceleration)
- \( g \) = Gravitational constant, 32.2 ft/sec\(^2\)

15. Distance traveled after accel/decelerating from one velocity to another over a unit of time, ft.

\[ d = 0.5T(Vo + Vf) \]

- \( d \) = Distance traveled, ft
- \( Vo \) = Velocity initial, ft/sec
- \( Vf \) = Velocity final, ft/sec
- \( T \) = Time, sec

16. Distance traveled after accelerating from one velocity to another with a known acceleration factor, ft.

\[ d = (Vf^2 - Vo^2) / 2fg \]

- \( d \) = Distance traveled, ft
- \( Vf \) = Velocity final, ft/sec
- \( Vo \) = Velocity initial, ft/sec
- \( f \) = Acceleration factor, decimal
- \( g \) = Gravitational constant, 32.2 ft/sec\(^2\)
17. Distance traveled after accelerating from one velocity to another with a known acceleration rate, \( \text{ft} \).

\[
d = \frac{(V_f^2 - V_o^2)}{2a}
\]

\( V_f = \) Velocity final, \( \text{ft/sec} \)

\( V_o = \) Velocity initial, \( \text{ft/sec} \)

\( a = \) Acceleration rate, \( \text{ft/sec}^2 \)

18. Distance traveled after accelerating from one speed to another with a known acceleration factor, \( \text{ft} \).

\[
d = \frac{(S_f^2 - S_o^2)}{30f}
\]

\( S_f = \) Speed final, \( \text{mi/hr} \)

\( S_o = \) Speed initial, \( \text{mi/hr} \)

\( f = \) Acceleration factor, decimal

19. Distance traveled after decelerating from one velocity to another with a known deceleration factor, \( \text{ft} \).

\[
d = \frac{(V_o^2 - V_f^2)}{2fg}
\]

\( V_o = \) Velocity initial, \( \text{ft/sec} \)

\( V_f = \) Velocity final, \( \text{ft/sec} \)

\( f = \) Deceleration factor, decimal

\( g = \) Gravitational constant, 32.2 \( \text{ft/sec}^2 \)

20. Distance traveled after decelerating from one velocity to another with a known deceleration rate, \( \text{ft} \).

\[
d = \frac{(V_o^2 - V_f^2)}{2a}
\]

\( V_o = \) Velocity initial, \( \text{ft/sec} \)

\( V_f = \) Velocity final, \( \text{ft/sec} \)

\( a = \) Deceleration rate, \( \text{ft/sec}^2 \)
21. Distance traveled after decelerating from one speed to another with a known deceleration factor, \( \text{ft} \).

\[
d = \left( \frac{So^2 - Sf^2}{30f} \right) / So = \text{Speed initial, mi/hr} \\
Sf = \text{Speed final, mi/hr} \\
f = \text{Deceleration factor, decimal}
\]

22. Distance a vehicle will move each tire revolution, \( \text{ft} \).

\[
d = r2\pi \\
r = \text{Tire radius, ft} \\
\pi = \text{Pi, 3.141592654}
\]

23. Distance at which headlight illumination can be considered as a spatial analogue to twilight, \( \text{ft} \). Combine the candelas output for each headlight prior to utilizing this formula.

\[
T_d = \sqrt{cd / 0.3} \\
cd = \text{Luminous intensity of headlights in candelas, cd}
\]

SAE 890684

1. Rolling resistance coefficient for bias or radial tires, \text{decimal}.

\[
f_{\text{roll}} = a + \frac{0.15}{p} + \frac{b}{p} (S / 100)^2 \\
S = \text{Speed, mi/hr} \\
p = \text{Tire inflation pressure, psi} \\
Limpert \\
Radial: \quad a = 0.005 \\
\quad b = 0.67 \\
Bias Ply: \quad a = 0.009 \\
\quad b = 1.0
\]
Energy

The capacity of a physical system to do work.

Acceleration / Deceleration, Distance

1. \[ E = Wfd \]
   Kinetic energy generated or dissipated while accel/decelerating over a determined distance, \textbf{ft-lb}.
   
   \( W = \) Total static weight, \( \text{lb} \)
   \( f = \) Accel / Decel factor, decimal
   \( d = \) Distance, \( \text{ft} \)

2. \[ E = W(\mu n \pm m)d \]
   Kinetic energy generated or dissipated while accel/decelerating over a determined distance, \textbf{ft-lb}.
   
   \( W = \) Total static weight, \( \text{lb} \)
   \( f = \) Accel / Decel factor, decimal
   \( d = \) Distance, \( \text{ft} \)
   \( n = \) Braking efficiency, decimal
   (deceleration only)
   \( m = \) Grade, Maximum 11.9\%, decimal
   \{(+ \text{ for incline}, (- \text{ for decline}) \}

3. \[ E = amd \]
   Kinetic energy generated or dissipated while accel/decelerating over a determined distance, \textbf{ft-lb}.
   
   \( a = \) Accel / Decel rate, \( \text{ft/sec}^2 \)
   \( m = \) Mass, \( \text{lb-sec}^2/\text{ft} \)
   \( d = \) Distance, \( \text{ft} \)
contained in motion

4. \( E = Fd \)  
   Energy contained in motion knowing the force applied over a determined distance, \textbf{ft-lb}.
   \[
   F = \text{Force, lb} \quad d = \text{Distance, ft}
   \]

5. \( E = \frac{S^2W}{30} \)  
   Energy contained in motion knowing the speed and weight, \textbf{ft-lb}.
   \[
   S = \text{Speed, mi/hr} \quad W = \text{Total static weight, lb}
   \]

6. \( E = 0.5mV^2 \)  
   Kinetic energy contained in motion knowing the velocity and mass, \textbf{ft-lb}.
   \[
   m = \text{Mass, lb-sec}^2/\text{ft} \quad V = \text{Velocity, ft/sec}
   \]

7. \( E = \frac{WV^2}{2g} \)  
   Kinetic energy contained in motion knowing the velocity and weight, \textbf{ft-lb}.
   \[
   V = \text{Velocity, ft/sec} \quad W = \text{Total static weight, lb} \quad g = \text{Gravitational constant, 32.2 ft/sec}^2
   \]
**Acceleration / Deceleration, One Velocity to Another**

8. \[ E = \frac{m}{2}(V_f^2 - V_o^2) \]

Kinetic energy generated or dissipated during an accel/deceleration from one velocity to another, **ft-lb**.

(negative solution (-) for deceleration)

\[ m = \text{Mass, lb-sec}^2/\text{ft} \]
\[ V_o = \text{Velocity initial, ft/sec} \]
\[ V_f = \text{Velocity final, ft/sec} \]

**Applied Energy**

9. \[ E = Fd\cos\theta \]

Energy applied to an object by a force at an angle, **ft-lb**.

\[ F = \text{Force, lb} \]
\[ d = \text{Distance, ft} \]
\[ \theta = \text{Angle between force applied and direction the object is moved, deg} \]

**Rotational Kinetic Energy**

10. \[ E = 0.5I\omega^2 \]

Rotational kinetic energy of an object, **ft-lb**.

\[ I = \text{Mass moment of inertia, ft-lb-sec}^2 \]
\[ \omega = \text{Angular velocity, rad/sec} \]

**Limpert**

\[ I = \text{Eq #5 Mass section} \]
\[ \omega = \text{Eq #14 Damage (Crush) section} \]

11. \[ E = 0.00873Wf\ell\alpha \]

Rotational kinetic energy of an object, **ft-lb**.

\[ W = \text{Total static weight, lb} \]
\[ f = \text{Rotational friction coefficient, decimal} \]
\[ \ell = \text{Wheelbase, ft} \]
\[ \alpha = \text{Post-impact angle after rotation, deg} \]
**Gravitational Potential Energy**

12. \[ E_p = Wh \]

Gravitational potential energy of an object knowing the weight and vertical height raised above a reference plane, *ft-lb*.

\( W \) = Total static weight, lb
\( h \) = Vertical height raised above a reference plane, ft

**Combined Energy**

13. \[ Ec = \left( \frac{m_1}{2} \right) \left( V_1^2 - V_3^2 \right) + \left( \frac{m_2}{2} \right) \left( V_2^2 - V_4^2 \right) \]

Combined energy generated during a two vehicle impact (linear momentum), *ft-lb*.

\( m_1 \) = Mass, vehicle #1, lb-sec^2/ft
\( m_2 \) = Mass, vehicle #2, lb-sec^2/ft
\( V_1 \) = Pre-impact velocity veh #1, ft/sec
\( V_2 \) = Pre-impact velocity veh #2, ft/sec
\( V_3 \) = Post-impact velocity veh #1, ft/sec
\( V_4 \) = Post-impact velocity veh #2, ft/sec

14. \[ Ec = 0.0334 \left[ W_1 \left( S_1^2 - S_3^2 \right) + W_2 \left( S_2^2 - S_4^2 \right) \right] \]

Combined energy generated during a two vehicle impact (linear momentum), *ft-lb*.

\( W_1 \) = Weight, vehicle #1, lb
\( W_2 \) = Weight, vehicle #2, lb
\( S_1 \) = Pre-impact speed veh #1, mi/hr
\( S_2 \) = Pre-impact speed veh #2, mi/hr
\( S_3 \) = Post-impact speed veh #1, mi/hr
\( S_4 \) = Post-impact speed veh #2, mi/hr
Dissipation of Energy

15. \[ V_1^2 W_1 + V_2^2 W_2 = V_3^2 W_1 + V_4^2 W_2 + bev_1^2 W_1 + bev_2^2 W_2 \]
\[ V_1^2 W_1 + V_2^2 W_2 = V_3^2 W_1 + I_1 g \omega_1^2 + I_2 g \omega_1^2 + V_4^2 W_2 + bev_1^2 W_1 + bev_2^2 W_2 \]

Dissipation of energy, \text{ ft/sec}. \\

\[ W_1 = \text{Weight, vehicle #1, lb} \]
\[ W_2 = \text{Weight, vehicle #2, lb} \]
\[ V_1 = \text{Pre-impact velocity veh #1, ft/sec} \]
\[ V_2 = \text{Pre-impact velocity veh #2, ft/sec} \]
\[ V_3 = \text{Post-impact velocity veh #1, ft/sec} \]
\[ V_4 = \text{Post-impact velocity veh #2, ft/sec} \]
\[ bev_1 = \text{Barrier equivalent velocity for vehicle #1, ft/sec} \]
\[ (\text{Eq #8 Crush Damage section}) \]
\[ bev_2 = \text{Barrier equivalent velocity for vehicle #2, ft/sec} \]
\[ (\text{Eq #8 Crush Damage section}) \]
\[ g = \text{Gravitational constant, 32.2 ft/sec} \]
\[ \omega_1 = \text{Angular velocity Vel #1, rad/sec} \]
\[ (\text{Eq #14 Damage (Crush) section}) \]
\[ \omega_2 = \text{Angular velocity Vel #2, rad/sec} \]
\[ (\text{Eq #14 Damage (Crush) section}) \]
\[ I_1 = \text{Yaw moment of Inertia, ft-lb-sec}^2 \]
\[ I_2 = \text{Yaw moment of Inertia, ft-lb-sec}^2 \]
16. \[ V_1 = \sqrt{V_3^2 + bev_1^2 + \frac{V_4^2W_2 + bev_2^2W_2 - V_1^2W_1}{W_1}} \]

Dissipation of energy for vehicle #1, \textit{ft/sec}.

\[ W_1 = \text{Weight, vehicle } #1, \text{ lb} \]
\[ W_2 = \text{Weight, vehicle } #2, \text{ lb} \]
\[ V_2 = \text{Pre-impact velocity veh } #2, \text{ ft/sec} \]
\[ V_3 = \text{Post-impact velocity veh } #1, \text{ ft/sec} \]
\[ V_4 = \text{Post-impact velocity veh } #2, \text{ ft/sec} \]
\[ bev_1 = \text{Barrier equivalent velocity for vehicle } #1, \text{ ft/sec} \]
\[ \text{(Eq #8 Crush Damage section)} \]
\[ bev_2 = \text{Barrier equivalent velocity for vehicle } #2, \text{ ft/sec} \]
\[ \text{(Eq #8 Crush Damage section)} \]

17. \[ V_2 = \sqrt{V_4^2 + bev_2^2 + \frac{V_3^2W_1 + bev_1^2W_1 - V_2^2W_2}{W_2}} \]

Dissipation of energy for vehicle #2, \textit{ft/sec}.

\[ W_1 = \text{Weight, vehicle } #1, \text{ lb} \]
\[ W_2 = \text{Weight, vehicle } #2, \text{ lb} \]
\[ V_1 = \text{Pre-impact velocity veh } #1, \text{ ft/sec} \]
\[ V_3 = \text{Post-impact velocity veh } #1, \text{ ft/sec} \]
\[ V_4 = \text{Post-impact velocity veh } #2, \text{ ft/sec} \]
\[ bev_1 = \text{Barrier equivalent velocity for vehicle } #1, \text{ ft/sec} \]
\[ \text{(Eq #8 Crush Damage section)} \]
\[ bev_2 = \text{Barrier equivalent velocity for vehicle } #2, \text{ ft/sec} \]
\[ \text{(Eq #8 Crush Damage section)} \]
**Speed**

18. $S = \sqrt{\frac{30E}{W}}$

   Determine a speed knowing the kinetic energy generated and weight of the object, \textit{mi/hr}.

   $E = \text{Kinetic energy, \text{ft-lb}}$
   $W = \text{Total static weight, \text{lb}}$

**Velocity**

19. $V = \sqrt{\frac{2gE}{W}}$

   Determine a velocity knowing the kinetic energy generated and weight of the object, \textit{ft/sec}.

   $E = \text{Kinetic energy, \text{ft-lb}}$
   $W = \text{Total static weight, \text{lb}}$
   $g = \text{Gravitational constant, 32.2 \text{ft/sec}^2}$

20. $V_o = \sqrt{\frac{V_f^2 - 2gE}{W}}$

   Initial velocity of an accel/deceleration knowing the kinetic energy generated or dissipated and final velocity, \textit{ft/sec}.

   $V_f = \text{Velocity final, \text{ft/sec}}$
   $E = \text{Kinetic energy, \text{ft-lb}}$
   $W = \text{Total static weight, \text{lb}}$
   $g = \text{Gravitational constant, 32.2 \text{ft/sec}^2}$

21. $V_f = \sqrt{\frac{V_o^2 + 2gE}{W}}$

   Final velocity of an accel/deceleration knowing the kinetic energy generated or dissipated and initial velocity, \textit{ft/sec}.

   $V_o = \text{Velocity initial, \text{ft/sec}}$
\[ E = \text{Kinetic energy, ft-lb} \]
\[ (\text{negative value (-) for deceleration}) \]
\[ W = \text{Total static weight, lb} \]
\[ g = \text{Gravitational constant, } 32.2 \text{ ft/sec}^2 \]

**Equivalents**

*Capable of being put into a one-to-one relationship. Used of two sets.*

**Inches to Tenths of a Foot**

<table>
<thead>
<tr>
<th>Inches</th>
<th>Tenths of a Foot</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.08 ft</td>
</tr>
<tr>
<td>2</td>
<td>.17 ft</td>
</tr>
<tr>
<td>3</td>
<td>.25 ft</td>
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<tr>
<td>4</td>
<td>.33 ft</td>
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<td>5</td>
<td>.42 ft</td>
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<td>.50 ft</td>
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<td>.75 ft</td>
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<td>10</td>
<td>.83 ft</td>
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<tr>
<td>11</td>
<td>.92 ft</td>
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<tr>
<td>12</td>
<td>1.0 ft</td>
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**Distance**

<table>
<thead>
<tr>
<th>Distance</th>
<th>Metric</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Foot</td>
<td>12 inches</td>
</tr>
<tr>
<td>1 Yard</td>
<td>3 Feet</td>
</tr>
<tr>
<td>1 Rod</td>
<td>5.5 Yards</td>
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<tr>
<td>1 Mile</td>
<td>1760 Yards</td>
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**Imperial/Metric**

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<td></td>
<td>0.254 decimeters</td>
</tr>
<tr>
<td></td>
<td>0.0254 meters</td>
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<tr>
<td>Measurement</td>
<td>Conversion to Meter</td>
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<tr>
<td>-------------</td>
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<tr>
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</tr>
<tr>
<td></td>
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<tr>
<td></td>
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<tr>
<td></td>
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<tr>
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<tr>
<td></td>
<td>91.44 cm</td>
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<tr>
<td></td>
<td>9.144 dm</td>
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<tr>
<td></td>
<td>0.9144 m</td>
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<tr>
<td>1 Mile</td>
<td>1609.3 m</td>
</tr>
<tr>
<td></td>
<td>1.6093 km</td>
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</table>

**Force**

A vector quantity that tends to produce an acceleration of a body in the direction of its application.

1. \[ F = ma \]

   Applied force knowing an object’s mass and accel/deceleration rate, \( \text{lb} \). Newton’s second law.

   \[ m = \text{Mass, } \text{lb}-\text{sec}^2/\text{ft} \]
   \[ a = \text{Accel / Decel rate, } \text{ft/sec}^2 \]
2. \[ F = Wf \]
   Required force to accel/decelerate (move) an object, \( \text{lb} \).

   \( W \) = Total static weight, \( \text{lb} \)
   \( f \) = Accel / Decel factor, \( \text{decimal} \)

3. \[ F = \frac{WS^2}{30d} \]
   Applied force during a deceleration to a stop, \( \text{lb} \).

   \( S \) = Speed, \( \text{mi/hr} \)
   \( d \) = Distance, \( \text{ft} \)
   \( W \) = Total static weight, \( \text{lb} \)

G. \[ F = \frac{WV^2}{2gd} \]
   Applied force during a deceleration to a stop, \( \text{lb} \).

   \( V \) = Velocity, \( \text{ft/sec} \)
   \( d \) = Distance, \( \text{ft} \)
   \( W \) = Total static weight, \( \text{lb} \)
   \( g \) = Gravitational constant, \( 32.2 \text{ ft/sec}^2 \)

4. \[ F = \left( A + BC_{\text{ave}} \right) \frac{L}{\cos \sigma} \]
   Collision force at the centroid of damage utilizing stiffness coefficients, \( \text{lb} \).

   \( A \) = Constant (stiffness coefficient), \( \text{lb/in} \)
   \( B \) = Constant (stiffness coefficient), \( \text{lb/in}^2 \)
   \( L \) = Width of crush region, \( \text{in} \)
   \( \sigma \) = Angle of attack at impact, \( \text{deg} \)
   (angle between the PDOF (\( \rho \)) and the damaged side axis) Do not exceed 45 degrees.

   \( C_{\text{ave}} \) = Weighted average crush of vehicle, \( \text{in} \)
   (Equally spaced points)
5. \( F_E = F \cos \rho_1 \)  
   Effective force which is parallel to the direction of travel for vehicle #1, \text{lb}.

   \( F = \text{Force, \ lb (Eq \#4)} \)
   \( \rho_1 = \text{Principal direction of force, \ deg} \)
   \( \text{(Eq \#14 Momentum section)} \)

6. \( F_E = F \sin \rho_2 \)  
   Effective force which is parallel to the direction of travel for vehicle #2, \text{lb}.

   \( F = \text{Force, \ lb (Eq \#4)} \)
   \( \rho_2 = \text{Principal direction of force, \ deg} \)
   \( \text{(Eq \#15 Momentum section)} \)

7. \( F_a = y_c \cos \rho - x_c \sin \rho \)  
   Determine the offset (moment arm of force) from the center of gravity to the line of force applied through the centroid of damage, knowing the center of gravity coordinates to the centroid of damage, \text{in}. Positive value for clockwise rotation to the vehicle.

   \( x_c = \text{Distance from center of gravity to centroid of damage in the} \) 
   \( x \text{ direction, \ in} \)
   \( \text{(Damage Centroid section)} \)
   \( y_c = \text{Distance from center of gravity to centroid of damage in the} \) 
   \( y \text{ direction, \ in} \)
   \( \text{(Damage Centroid section)} \)
   \( \rho = \text{Principal direction of force, \ deg} \)
   \( \text{(Eq \#14, 15 Momentum section)} \)
8. \[ F_{\text{cent}} = a_y \frac{W}{g} \] Centrifugal and centripetal forces in a turn when they are in balance, \textbf{lb}. Newton's third law.

\[ a_y = \text{Lateral acceleration rate, } \text{ft/sec}^2 \]
\[ W = \text{Total static weight, } \text{lb} \]
\[ g = \text{Gravitational constant, } 32.2 \text{ ft/sec}^2 \]

9. \[ F_{\text{cent}} = \left( \frac{V^2}{r} \right) \frac{W}{g} \] Centrifugal (inertia) and centripetal forces in a turn when they are in balance, \textbf{lb}. Newton's third law. Sideslip will commence if the inertial value exceeds the centripetal force (Eq 10 – 11).

\[ V = \text{Velocity, } \text{ft/sec} \]
\[ r = \text{Turning radius, } \text{ft} \]
\[ W = \text{Total static weight, } \text{lb} \]
\[ g = \text{Gravitational constant, } 32.2 \text{ ft/sec}^2 \]

10. \[ F_y = W \mu \cos \alpha \] Cornering force applied to a tire which is at its frictional limit, \textbf{lb}.

\[ \mu = \text{Friction coefficient, decimal} \]
\[ \alpha = \text{Tire slip angle, } \text{deg} \]
\[ W = \text{Weight on tire, } \text{lb} \]

11. \[ F_{\text{br}} = W \mu \sin \alpha \] Braking force applied to a tire, which is at its frictional limit, \textbf{lb}.

\[ \mu = \text{Friction coefficient, decimal} \]
\[ \alpha = \text{Tire slip angle, } \text{deg} \]
\[ W = \text{Weight on tire, } \text{lb} \]
12. \( F_{yF} = F_{cent} x_{Ri} \)

Frictional cornering force applied to the front axle in a turn, \( \text{lb} \).

\( F_{cent} \) = Centrifugal force, \( \text{lb} \) (Eq #8, 9)

\( x_{Ri} \) = Longitudinal center of mass measured from the rear axle as a decimal fraction of the wheelbase \( (x_{R} / \ell) \), decimal

(Eq #4 Center of Mass section)

13. \( F_{yF} = W V^2 x_{Ri} / (g r) \)

Frictional cornering force applied to the front axle in a turn, \( \text{lb} \).

\( V \) = Velocity, \( \text{ft/sec} \)

\( r \) = Radius traveled by center of mass, \( \text{ft} \)

\( W \) = Total static weight, \( \text{lb} \)

\( g \) = Gravitational constant, \( 32.2 \text{ ft/sec}^2 \)

\( x_{Ri} \) = Longitudinal center of mass measured from the rear axle as a decimal fraction of the wheelbase \( (x_{R} / \ell) \), decimal

(Eq #4 Center of Mass section)

14. \( F_{yR} = F_{cent} x_{Fi} \)

Frictional cornering force applied to the rear axle in a turn, \( \text{lb} \).

\( F_{cent} \) = Centrifugal force, \( \text{lb} \) (Eq #8, 9)

\( x_{Fi} \) = Longitudinal center of mass measured from the front axle as a decimal fraction of the wheelbase \( (x_{F} / \ell) \), decimal

(Eq #2 Center of Mass section)
15. \[ F_{cr} = \frac{WV^2 x_f}{(gr)} \] Frictional cornering force applied to the rear axle in a turn, \text{lb}.

\[
V = \text{Velocity, ft/sec} \\
r = \text{Radius traveled by center of mass, ft} \\
W = \text{Total static weight, lb} \\
g = \text{Gravitational constant, 32.2 ft/sec}^2 \\
x_{f_i} = \text{Longitudinal center of mass measured from the front axle as a decimal fraction of the wheelbase (} x_f / \ell \text{), decimal} \\
\text{(Eq #2 Center of Mass section)}
\]

**Grade & Superelevation**

*A slope or gradual inclination, especially of a road or railroad track.*

1. Grade or superelevation of a surface to maximum of 11.9\%, \text{pct.}

\[
e = \left(\frac{h}{d}\right) 100 \\
m = \left(\frac{h}{d}\right) 100
\]

\[
h = \text{Vertical height (rise), ft} \\
d = \text{Horizontal distance (run), ft}
\]

2. Percent of a grade or superelevated surface knowing the degree, \text{pct.}

\[
m = \tan \theta (100) \\
e = \tan \theta (100)
\]

\[
\theta = \text{Grade or superelevated surface, deg (Eq #5, 7)}
\]

3. Vertical height (rise) of a grade or super-elevated surface knowing the horizontal distance (run), \text{ft.}

\[
h = \frac{md}{100} \\
h = \frac{ed}{100}
\]

\[
d = \text{Horizontal distance (run), ft} \\
m = \text{Grade, maximum 11.9\%, pct (Eq #2)} \\
e = \text{Superelevation, pct (Eq #2)}
\]
4. Horizontal distance (run) of a grade or superelevated surface knowing the vertical height (rise), ft.

\[ d = \frac{h}{e} \times 100 \]
\[ d = \frac{h}{m} \times 100 \]

- \( h \) = Vertical height (rise), ft
- \( m \) = Grade, maximum 11.9%, pct (Eq #2)
- \( e \) = Superelevation, pct (Eq #2)

5. Degrees of a grade or superelevated surface to a maximum of 6.8°, deg.

\[ \theta = \tan^{-1}\left(\frac{h}{d}\right) \]

- \( h \) = Vertical height (rise), ft (Eq #3)
- \( d \) = Horizontal distance (run), ft (Eq #4)

6. Degrees of a grade or superelevated surface to a maximum of 6.8°, deg.

\[ \theta = \tan^{-1}\left(\frac{e}{100}\right) \]
\[ \theta = \tan^{-1}\left(\frac{m}{100}\right) \]

- \( m \) = Grade, maximum 11.9%, pct (Eq #1)
- \( e \) = Superelevation, pct (Eq #1)

7. Degrees of a grade or superelevated surface to a maximum of 6.8°, deg.

\[ \theta = \frac{\delta}{180}/\pi \]

- \( \delta \) = Grade, maximum 0.1186, rad
- \( \pi \) = Pi, 3.141592654

8. Radians of a grade or superelevated surface to a maximum of 0.1186 radians, rad.

\[ \delta = \frac{\theta\pi}{180} \]

- \( \theta \) = Grade, 6.8° maximum, deg (Eq #5, 6)
- \( \pi \) = Pi, 3.141592654

9. Equivalent friction coefficient of a level surface from a grade, decimal.

\[ \mu_e = \mu - \sin(\tan^{-1}(m))/\cos(\tan^{-1}(m)) \]

- \( \mu \) = Friction coefficient of grade, decimal
- \( m \) = Grade, decimal (negative value (-) for decline)
10. Equivalent friction coefficient of a grade from a level surface, decimal.

$$\mu_e = \sin(Tan^{-1}(m)) + \mu \cdot \cos(Tan^{-1}(m)) \quad \mu = \text{Level friction coefficient, decimal}$$

$$m = \text{Grade, decimal (negative value (-) for decline)}$$

11. Equivalent friction coefficient of a grade from a level surface, decimal.

$$\mu_e = \frac{\mu \pm m}{\sqrt{1 \pm m^2}} \quad \mu = \text{Level friction coefficient, decimal}$$

$$m = \text{Grade, decimal \{(+ for incline, -) for decline\}}$$

12. Velocity calculated for a vehicle sliding laterally in an original forward movement due to roadway grade, ft/sec.

$$V = 2f \sqrt{\frac{gd_y}{\sin \alpha}} \quad f = \text{Deceleration factor, decimal}$$

$$d_y = \text{Lateral distance, ft}$$

Limpert

$$\alpha = \text{Lateral or cross slope angle, deg (10 degrees or less)}$$

$$g = \text{Gravitational constant, 32.2 ft/sec}^2$$

Gravity

Any two bodies in the universe attract each other with a force that is directly proportional to their masses and inversely proportional to the square of their distance apart.

R.W. Rivers

Acceleration of Gravity

1. Acceleration of gravity knowing the weight and mass of an object, ft/sec².

$$g = \frac{W}{m} \quad W = \text{Total static weight, lb}$$

$$m = \text{Mass, lb-sec}^2 /\text{ft}$$
2. Acceleration of gravity knowing the accel/deceleration rate and accel/deceleration factor of an object, \( \text{ft/sec}^2 \).

\[
g = \frac{a}{f}
\]

\( a = \) Accel / Decel rate, \( \text{ft/sec}^2 \)

\( f = \) Accel / Decel factor, decimal

**Gravitational potential Energy**

3. Gravitational potential energy of an object knowing the weight and vertical height raised above a reference plane, \( \text{ft-lb} \).

\[
E_p = Wh
\]

\( W = \) Total static weight, lb

\( h = \) Vertical height raised above a reference plane, ft

**Heavy Truck Impact**

**Equivalent Deceleration Factor; Tractor/Semi Trailer**

The variables for equations #1 through 6 of this chapter are defined as follows:

**Tractor:**

\( W = \) Total static tractor weight, lb

\( x_{Fi} = \) Longitudinal center of mass from the front axle as a decimal fraction of the wheelbase, decimal

(Eq #3 Center of Mass section)

\( x_h = \) Center of hitch (fifth wheel) longitudinal distance ahead of the tractor’s rear axle as a decimal fraction of the wheelbase, decimal (if the hitch is behind the rear axle, value is negative)

\( z_i = \) Vertical center of mass height as a decimal fraction of the wheelbase, decimal

(Eq #11 Center of Mass section)

\( z_h = \) Center of hitch vertical height as a decimal fraction of the tractor’s wheelbase, decimal

\( f_F = \) Tractor’s front axle deceleration factor, decimal

\( f_R = \) Tractor’s rear axle(s) deceleration factor, decimal
Semi trailer:

\[ W'' = \text{Total static weight of semi trailer and load, lb} \]
\[ x'_i = \text{Center of hitch longitudinal distance to combined center of mass of load and trailer as a decimal fraction of the semi trailer’s wheelbase} \left( \frac{x'}{\ell} \right), \text{decimal} \ (x' = \text{Eq \#22}) \]
\[ z'_i = \text{Center of mass vertical height of combined load and trailer as a decimal fraction of the wheelbase} \left( \frac{z'}{\ell} \right), \text{decimal} \ (z' = \text{Eq \#23}) \]
\[ z'_h = \text{Center of hitch vertical height as a decimal fraction of the wheelbase, decimal} \]
\[ f_i = \text{Semi trailer rear axle(s) deceleration factor, decimal} \]

**Note:** Wheelbase \( \left( \ell \right) \) of the semi trailer is measured from the center of hitch to the center of the rear axle(s).

1. Equivalent deceleration factor for tractor and semi trailer, **decimal.**

\[ f_e = \frac{B + Kf_F + f_R\left(D + W - K\right)}{W + A - J\left(f_F - f_R\right) - Ef_R} \]

1a. \[ A = W\left[1 + \frac{f''(z'_i - z'_h)}{1 + z'_h f''} \right] \]

1b. \[ B = \frac{Wx'_i f'}{1 + z'_h f''} \]

1c. \[ D = W\left(1 - \frac{x'_i}{1 + z'_h f''} \right) \]

1d. \[ E = \frac{W'(z'_i - z'_h)}{1 + z'_h f''} \]

1e. \[ J = Wz_i + Ex_h + Az_h \]

1f. \[ K = W\left(1 - x_{Fi}\right) + Dx_h - Bz_h \]

**Force & Load During Braking; Tractor/Semi Trailer**

The variables for Eq \#2 through 6 are listed in above sub-section (1).

2. Load on the tractor's front axle during braking, **lb.**
\[ W_F = f_e J + K \]

3. Load on the tractor’s rear axle during braking, \textbf{lb}.
\[ W_R = f'_e (E - J) + W + D - K \]

4. Load on the trailer’s axle during braking, \textbf{lb}.
\[ W'_R = \frac{W'X'_i}{1 + z_h'f'_i} - f_e E \]

5. Vertical load of trailer on tractor’s hitch during braking, \textbf{lb}.
\[ V_Z = f_e E + D \]

6. Horizontal force of the trailer on the hitch during braking, \textbf{lb}.
\[ F_X = f_e A + B \]

**Equivalent Deceleration Factor; Powered Vehicle/Full trailer**

The variables for equations #7 through 12 of this chapter are defined as follows:

**Powered Vehicle:**

\( W \) = Total static weight of the powered vehicle, \textbf{lb}
\( x_{F_i} \) = Longitudinal center of mass from the front axle as a decimal fraction of the wheelbase, decimal
(Eq #3 Center of Mass section)
\( z_i \) = Vertical center of mass height as a decimal fraction of the wheelbase, decimal
\( z_h \) = Tongue hitch vertical height as a decimal fraction of the wheelbase, decimal
\( f_F \) = Powered vehicle’s front axle deceleration factor, decimal
\( f_R \) = Powered vehicle’s rear axle(s) deceleration factor, decimal
Full Trailer or Towed Vehicle:

\[ W' = \text{Total static weight of the towed vehicle/trailer, lb} \]

\[ x'_f = \text{Longitudinal center of mass from the front axle as a decimal fraction of the towed vehicle/trailer’s wheelbase} \left( \frac{x'}{\ell} \right), \text{decimal} \ (x' = \text{Eq #22}) \]

\[ z'_f = \text{Center of mass vertical height of combined load and towed vehicle/trailer as a decimal fraction of the wheelbase} \left( \frac{z'}{\ell} \right), \text{decimal} \ (z' = \text{Eq #23}) \]

\[ z'_h = \text{Center of hitch vertical height on the towed vehicle/trailer as a decimal fraction of the wheelbase}, \text{decimal} \]

\[ f'_f = \text{Towed vehicle/trailer’s front axle deceleration factor, decimal} \]

\[ f'_R = \text{Towed vehicle/trailer’s rear axle(s) deceleration factor, decimal} \]

7. Equivalent deceleration factor for the powered vehicle and towed vehicle/trailer, decimal.

\[
f_e = \left( \frac{W x_f - L}{z_h} + \frac{W x'_f - T}{z'_h} \right) - \frac{N}{z'_h + U} + \frac{W z'_f}{z'_h} \]

7a. \[ N = \frac{W (x'_h - z'_f)}{1 - z'_h (f'_f - f'_R)} \]

7b. \[ T = \frac{W (x'_f - f'_f z'_h)}{1 - z'_h (f'_f - f'_R)} \]

7c. \[ U = \frac{W (z'_h - z'_f)}{1 - z'_h (f'_f - f'_R)} \]

7d. \[ L = \frac{W (x_f - f'_f z_h)}{1 - z_h (f'_f - f'_R)} \]

**Force & Load During Braking; Powered Vehicle/Full Trailer**

The variables for Eq #8 through 12 are listed in above sub-section (7).

8. Load on the powered vehicle’s front axle during braking, lb.

\[ W_f = W - f_e U - L \]

9. Load on the powered vehicle’s rear axle during braking, lb.

\[ W_h = f_e U + L \]

10. Load on the towed vehicle/trailer’s front axle during braking, lb.
\[ W'_f = W' - f_f N - T \]

11. Load on the towed vehicle/trailer’s rear axle during braking, \textbf{lb}.

\[ W'_{TR} = f_f N + T \]

12. Horizontal force of the trailer applied to the hitch during braking, \textbf{lb}.

\[ F_x = \frac{f_f (N + W'_{z'}) + T - W'_{x'}}{z_h} \]

**Velocity; Weight Shift (articulated)**

The variables for equations #13 through 13j of this chapter are defined as follows:

- \( V_f = \) Velocity final, \text{ ft/sec} \\
- \( d = \) Straight skid distance, \text{ ft} \\
- \( g = \) Gravitational constant, \text{32.2 ft/sec}^2

**Tractor:**

- \( W = \) Total static tractor weight, \text{lb} \\
- \( x_f = \) Longitudinal center of mass from the front axle, \text{ ft} (Eq #1, 2 Center of Mass section) \\
- \( x_h = \) Longitudinal distance from the tractor’s center of mass to the center of hitch (fifth wheel), \text{ ft} \\
- \( x_R = \) Longitudinal distance from the tractor’s center of mass \( (x_f) \) to the center of the tandem drive axles, \text{ ft} (Eq #4, 5 Center of Mass section) \\
- \( z = \) Vertical center of mass height of the tractor from the ground, \text{ ft} \\
  \hspace{0.5cm} (Eq #9, 10 Center of Mass section) \\
- \( z_h = \) Center of hitch vertical height from the ground, \text{ ft} \\
- \( f_f = \) Tractor’s front axle deceleration factor, \text{decimal} \\
- \( f_R = \) Tractor’s rear axle(s) deceleration factor, \text{decimal}

**Semi trailer:**

- \( W' = \) Total static weight of semi trailer and load, \text{lb} \\
- \( x' = \) Longitudinal distance from center of mass to combined center of mass of load and trailer, \text{ ft} \\
  \hspace{0.5cm} (Eq #22) \\
- \( x'_R = \) Longitudinal distance from the center of mass of combined load & trailer to the center of the trailer’s tandem axles, \text{ ft} \text{ m} \\
- \( z' = \) Vertical height of combined center of mass of load and trailer from the ground, \text{ ft} \\
  \hspace{0.5cm} (Eq #23)
\( f' = \) Semi trailer rear axle(s) deceleration factor, decimal

13. Velocity accounting for weight shift during a deceleration for an articulated vehicle, \( \text{ft/sec} \).

\[
V_o = \sqrt{V f + 2gd}\frac{W(N - P) + W'[M + K((T + U)/R) + J]}{W + W'(Q - KZ)}
\]

Lofgren

13a. \( J = x' f'\left( x' + x'_R + f_R z_h \right) \)

13b. \( K = \frac{f_R - f_F}{x_F + x_R + z(f_R - f_F)} \)

13c. \( M = \frac{x'_R}{x' + x'_R} \left( \frac{f_F x_R + f_R x_F + x_h(f_R - f_F)}{x_F + x_R} \right) \)

13d. \( N = \frac{f_F x_R + f_R x_F}{x_F + x_R} \)

13e. \( P = \frac{z(f_R - f_F)(f_F x_R + f_R x_F)}{(x_F + x_R)(x_F + x_R + z(f_R - f_F))} \)

13f. \( Q = \frac{x' + x'_R + z'(f' - f'_h) + f_R z_h}{x' + x'_R + z_h f'} \)

13g. \( R = (x_F + x_R)(x' + x'_R)(x' + x'_R + z_h f') \)

13h. \( T = x'_R z(x' + x'_R + z_h f') [f_F(x_h - x_R) - f_R(x_F + x_h)] \)

13i. \( U = x' f'(x_F + x_R) [z_h(x_h - x_R) - (z - z_h)(x' + x'_R) - f_R z_h z] \)

13j. \( Z = \frac{z'(x_h - x_R) + z'(x' + x'_R) + z_h(x_R - x - x'_R - x_h) + z' z'(f' - f_R) - z_h(z' f' - f_R z)}{x' + x'_R + z_h f'} \)
Velocity; Weight Shift (non articulated)

14. Speed accounting for weight shift during a deceleration for a non-articulated vehicle with axle pairs close together, \textbf{mi/hr}.

\[ S = \sqrt{30 \left[ f_{Fr}d_{Fr}(W_{Fr} + \Delta W / 2) + f_{Fl}d_{Fl}(W_{Fl} + \Delta W / 2) + f_{Rr}d_{Rr}(W_{Rr} - \Delta W / 2) + f_{Rl}d_{Rl}(W_{Rl} - \Delta W / 2) \right]} \]

- \(d_{Fr}\) = Right-front tire skid distance, ft
- \(d_{Fl}\) = Left-front tire skid distance, ft
- \(d_{Rr}\) = Right-rear tire skid distance, ft
- \(d_{Rl}\) = Left-rear tire skid distance, ft
- \(f_{Fr}\) = Right-front deceleration factor, decimal
- \(f_{Fl}\) = Left-front deceleration factor, decimal
- \(f_{Rr}\) = Right-rear deceleration factor, decimal
- \(f_{Rl}\) = Left-rear deceleration factor, decimal
- \(W\) = Total static weight, lb
- \(W_{Fr}\) = Static right-front axle weight, lb
- \(W_{Fl}\) = Static left-front axle weight, lb
- \(W_{Rr}\) = Static right-rear axle weight, lb
- \(W_{Rl}\) = Static left-rear axle weight, lb
- \(\Delta W\) = Weight shift to front axle, lb

(Eq #1, 2, 3 Weight Shift section)
15. Velocity accounting for weight shift during a deceleration on a non-articulated vehicle with axle pairs close together, \( \text{ft/sec} \).

\[
V = \sqrt{\frac{2g}{W} \left[ f_{Fr} d_{Fr} (W_{Fr} + \Delta W / 2) + f_{Fl} d_{Fl} (W_{Fl} + \Delta W / 2) + f_{Rr} d_{Rr} (W_{Rr} - \Delta W / 2) + f_{Rl} d_{Rl} (W_{Rl} - \Delta W / 2) \right]}
\]

\( d_{Fr} = \text{Right-front tire skid distance, ft} \)
\( d_{Fl} = \text{Left-front tire skid distance, ft} \)
\( d_{Rr} = \text{Right-rear tire skid distance, ft} \)
\( d_{Rl} = \text{Left-rear tire skid distance, ft} \)
\( f_{Fr} = \text{Right-front deceleration factor, decimal} \)
\( f_{Fl} = \text{Left-front deceleration factor, decimal} \)
\( f_{Rr} = \text{Right-rear deceleration factor, decimal} \)
\( f_{Rl} = \text{Left-rear deceleration factor, decimal} \)
\( W = \text{Total static weight, lb} \)
\( W_{Fr} = \text{Static right-front axle weight, lb} \)
\( W_{Fl} = \text{Static left-front axle weight, lb} \)
\( W_{Rr} = \text{Static right-rear axle weight, lb} \)
\( W_{Rl} = \text{Static left-rear axle weight, lb} \)
\( \Delta W = \text{Weight shift to front axle, lb} \)

(Eq #1, 2, 3 Weight Shift section)

\( g = \text{Gravitational constant, 32.2 ft/sec}^2 \)

**Velocity; Gear Position & Transmission Measurements**

16. Overall final gear ratio, 00:1.

\[ i = i_T \cdot i_{TA} \cdot i_A \]

\( i_T = \text{Transmission gear ratio, 00:1} \)
\( i_{TA} = \text{Auxiliary transmission ratio, 00:1} \)
\( i_A = \text{Axle ratio, 00:1 (Differential gear ratio)} \)

17. Determine the total revolutions per second of the drive wheels, rev/sec.

\[ r_i = \frac{\omega_e / 60}{i} \]

\( \omega_e = \text{Rotational speed of the engine, rev/min} \)
\( i = \text{Final drive-gear ratio, 00:1 (Eq #3, 4 RPM section)} \)

18. Determine the revolutions per minute of the drive wheels at impact, rev/min.
\[ \omega_{e_1} = \omega_e / i \]

\[ \omega_e = \text{Rotational speed of the engine, rev/min} \]
\[ i = \text{Final drive-gear ratio, 00:1 (Eq #3, 4 RPM section)} \]

19. Drive wheel revolutions per second knowing the rolling wheel radius, rev/sec.

\[ rps = 6.28318r \]

\[ r = \text{Tire radius, ft} \]

20. Speed from transmission measurements, mi/hr.

\[ S = \frac{0.00595 \omega_e * r}{i_T * i_{TA} * i_A} \]

\[ \omega_e = \text{Rotational speed of the engine, rev/min} \]
\[ r = \text{Tire radius, in} \]
\[ i_T = \text{Transmission gear ratio, 00:1} \]
\[ i_{TA} = \text{Auxiliary transmission ratio, 00:1} \]
\[ i_A = \text{Axle ratio, 00:1 (Differential gear ratio)} \]

21. \[ V = 0.00436r(\omega_e / (i_T * i_{TA} * i_A)) \] Velocity from transmission measurements, ft/sec.

\[ \omega_e = \text{Rotational speed of the engine, rev/min} \]
\[ r = \text{Tire diameter, in} \]
\[ i_T = \text{Transmission gear ratio, 00:1} \]
\[ i_{TA} = \text{Auxiliary transmission ratio, 00:1} \]
\[ i_A = \text{Axle ratio, 00:1} \]
\[ \text{(Differential gear ratio)} \]

22. Velocity from transmission measurements, ft/sec.
\[ V = \left[ \frac{\omega_e}{(i_T \times i_{TA} \times i_A)} \right] \left( r \frac{2\pi}{12} \right) / 60 \]

\( \omega_e \) = Rotational speed of the engine, rev/min
\( r \) = Tire radius, in
\( i_T \) = Transmission gear ratio, 00:1
\( i_{TA} \) = Auxiliary transmission ratio, 00:1
\( i_A \) = Axle ratio, 00:1 (Differential gear ratio)
\( \pi \) = Pi, 3.141592654

**Trailer; Center of Mass**

23. Longitudinal center of mass of combined trailer with load measured from a datum line, ft.

\[ x = \frac{W_L x_L + W_T x_T}{W} \]

\( x_L \) = Longitudinal distance from the datum line to center of mass of load, ft
\( x_T \) = Longitudinal distance from the datum line to the trailer’s center of mass, ft
\( W_L \) = Static weight of load, lb
\( W_T \) = Static weight of trailer, lb
\( W \) = Total static weight of semi trailer and load, lb

24. Lateral center of mass of combined trailer with load measured from a datum line, ft.

\[ y = \frac{W_L y_L + W_T y_T}{W} \]

\( y_L \) = Longitudinal distance from the datum line to center of mass of load, ft
\( y_T \) = Longitudinal distance from the datum line to the trailer’s center of mass, ft
\( W_L \) = Static weight of load, lb
\( W_T \) = Static weight of trailer, lb
\( W \) = Total static weight of semi trailer and load, lb

25. Vertical center of mass height of combined trailer with load, ft.
\[ z = \frac{W_L z_L + W_T z_T}{W} \]

- \( z_L \): Vertical center of mass height of load from the ground, ft
- \( z_T \): Vertical center of mass height of trailer, ft
- \( W_L \): Static weight of load, lb
- \( W_T \): Static weight of trailer, lb
- \( W \): Total static weight of semi trailer and load, lb

**Trailer Swing**

26. Angle at which a trailer will swing out with full rear axle lockup during braking, deg. The initial approach must be in a forward straight movement. If the trailer swings out 13-17 degrees, it may not be recoverable.

\[
\phi = \tan^{-1} \left( \frac{F_y x_a}{F_x (x_a + x_b) - Z x_a} \right)
\]

- \( F_x \): Drag force, ft-lb
- \( F_y \): Side Force, ft-lb
- \( x_a \): Center of hitch longitudinal distance to combined center of mass of load and trailer, ft
- \( x_b \): Rear axle longitudinal distance to combined center of mass of load and trailer, ft
- \( W_T \): Weight of trailer, lb
- \( W \): Weight of combined unit, lb
- \( W_A \): Weight on trailer rear axle, lb
- \( \mu \): Friction coefficient, decimal
- \( \theta \): Roadway pitch, deg

27. Distance a trailer will move laterally during a trailer swing out, ft.

\[
d_y = \ell_k \tan \phi
\]

- \( \ell_k \): Distance from the tractor’s rear axle to the 5th wheel kingpin, ft
- \( \phi \): Angle to which the trailer has swung out, deg (Eq #26)
Hydroplane

28. Speed required to hydroplane for commercial motor vehicles which is lightly laded, \textbf{mi/hr}. The depth of the water on the surface must exceed the tread depth of tires across any parallel point before a vehicle will hydroplane. There must be at least $\frac{1}{5}$ of an inch of water depth.

\[ S = 7.95\sqrt{\frac{p}{(w/L)^{-1}}} \]

\[ p = \text{Front-tire inflation pressure, psi} \]
\[ w = \text{Width of tire contact patch, in} \]
\[ L = \text{Length of tire contact patch, in} \]

Dunlap

29. Speed required to hydroplane for commercial motor vehicles which is lightly laded, \textbf{mi/hr}. The depth of the water on the surface must exceed the tread depth of tires across any parallel point before a vehicle will hydroplane. There must be at least $\frac{1}{5}$ of an inch of water depth.

\[ S = 27.40 p^{0.21} \sqrt{(w/L)^{-1}} \]

\[ p = \text{Front-tire inflation pressure, psi} \]
\[ w = \text{Width of tire contact patch, in} \]
\[ L = \text{Length of tire contact patch, in} \]

Ivey

Skip Skid Marks

30. Velocity from the measurements of skip skid marks, \textbf{ft/sec}. The road surface must be even and level.

\[ V = \sqrt{\frac{161 f(n-2)^2 d_o^2}{(n-2)d_o - d_r}} \]

Kwasnoski

\[ f = \text{Deceleration factor, decimal} \]
\[ n = \text{Number of skip skid marks, \#} \]
\[ d_o = \text{Distance between the first pair of marks, ft} \]
\[ d_r = \text{Distance between the middle of the first and last pairs of marks, ft} \]

Miscellaneous

31. Distance a truck will move each tire revolution, \textbf{ft}.

\[ d = r2\pi \]
\[ r = \text{Tire radius, ft} \]
\[ \pi = \text{Pi, 3.141592654} \]
32. Approximate weight supported by a wheel of a truck knowing the contact patch area of the tire on the road surface, \textbf{lb}.

\[ W = p(a) \]

\[ p = \text{Tire inflation pressure, psi} \]
\[ a = \text{Tire contact area with road surface, in}^2 \]

\textbf{Wind Speed Required, Rollover}

33. Theoretical wind speed required to cause wheel lift or rollover, \textbf{mi/hr}.

\[ S = \sqrt{(0.5Wtw)/(0.0025614z)} \]

\[ W = \text{Gross weight of vehicle, lb} \]
\[ tw = \text{Track width, ft} \]
\[ A = \text{area of windward side, ft}^2 \]
\[ z = \text{Vertical center of mass height, ft} \]

\textit{Ravensdale}

\[ \text{(Eq #7, 8 Center of Mass section)} \]

34. Coefficient of brake linings, \textbf{decimal}.

\[ C_f = \frac{T_{rr} \cdot 0.6W_i}{C_r D_r C_a I_i} \]

\[ T_{rr} = \text{Tire rolling radius, in} \]
\[ W_i = \text{Weight on wheel, lb} \]
\[ C_r = \text{Brake cam radius, in} \]
\[ D_r = \text{Drum radius, in} \]
\[ C_a = \text{Air chamber area, in} \]
\[ I_i = \text{Slack adjuster length, in (Pin to c/l)} \]
\[ p = \text{Air pressure at test (normally 60), psi} \]
35. Brake force at wheel, lb.

\[ B_f = \frac{2pC_aI_iC_fD_r}{C_rT_{rr}} \]

- \( C_f \) = Coefficient of brake lining, decimal (Eq #34)
- \( T_{rr} \) = Tire rolling radius, in
- \( C_r \) = Brake cam radius, in
- \( D_r \) = Drum radius, in
- \( C_a \) = Air chamber area, in
- \( I_i \) = Slack adjuster length, in (Pin to c/l)
- \( p \) = Air pressure at test (normally 90), psi

**Note:** To determine braking percentage of vehicle, add each wheel brake force together then divide by total weight of vehicle.

36. Rolling resistance coefficient for radial tires on heavy trucks, decimal.

\[ f_{roll} = (0.0041 + 0.000041V)f \]

- \( V \) = Velocity, ft/sec
- \( f \) = Friction coefficient, decimal

*University of Michigan*

- 1.0; smooth concrete
- 1.2; worn concrete, brick, cold blacktop
- 1.5; hot blacktop

37. Rolling resistance coefficient for bias-ply tires on heavy trucks, decimal.

\[ f_{roll} = (0.0066 + 0.000046V)f \]

- \( V \) = Velocity, ft/sec
- \( f \) = Friction coefficient, decimal

*University of Michigan*

- 1.0; smooth concrete
- 1.2; worn concrete, brick, cold blacktop
- 1.5; hot blacktop
**Linear Distance**

1. \[ d_x = \sqrt{4rd_y - d_y^2} \]

Linear distance required in a lane change maneuver knowing the radius of the turn and the lateral distance, \( \text{ft} \).

- \( d_y \) = Lateral turning distance, \( \text{ft} \)
- \( r \) = Radius traveled by center of mass, \( \text{ft} \)

**Lateral Distance**

2. \[ d_y = 0.5(r - r\cos\theta) \]

Lateral distance of a lane change knowing the radius of the turn and central angle traversed, \( \text{ft} \).

- \( r \) = Radius traveled by center of mass, \( \text{ft} \)
- \( \theta \) = Central angle traversed, \( \text{deg} \) (Eq #6)

3. \[ d_y = f_y \left( \frac{d_x}{0.366/S} \right)^2 \]

Lateral distance of a lane change at a constant speed knowing the linear distance and lateral acceleration factor, \( \text{ft} \).

- \( S \) = Speed, \( \text{mi/hr} \)
- \( f_y \) = Lateral acceleration factor, \( \text{decimal} \)
- \( d_x \) = Linear distance, \( \text{ft} \)

**Braking, Linear Distance**

N. \[ d_i = 0.458V \sqrt{\frac{d_y}{f_y} - 2n\mu (d_y / f_y)} \]

Linear distance required for a lane change while braking, \( \text{ft} \).

- \( V \) = Velocity, \( \text{ft/sec} \)
- \( \mu \) = Friction coefficient, \( \text{decimal} \)
- \( f_y \) = Lateral acceleration factor, \( \text{decimal} \)
- \( d_y \) = Lateral distance, \( \text{ft} \)
- \( n \) = Braking efficiency, \( \text{decimal} \)
4. \[ d_t = r2\pi(\theta / 360) \]

Distance traveled in the lane change knowing the central angle traversed and radius traveled by the center of mass, ft.

\( r \) = Radius traveled by center of mass, ft
\( \pi \) = Pi, 3.141592654
\( \theta \) = Central angle traversed, deg (Eq #6)

5. \[ d_t = \left(\frac{S}{1.93}\right)\sqrt{d_y / f_y} \]

Linear distance traveled in the lane change knowing the speed, lateral distance and lateral acceleration factor, ft.

\( S \) = Speed, mi/hr
\( f_y \) = Lateral acceleration factor, decimal
\( d_y \) = Lateral distance, ft

6. \[ \theta = \sin^{-1}\left(\frac{d_x}{r}\right) \]

Degree of the turn in a lane change maneuver knowing the radius and linear distance, deg.

\( d_x \) = Linear distance, ft
\( r \) = Radius traveled by center of mass, ft

7. \[ r = V^2 / a_y \]

Turning radius required to change lanes knowing the initial velocity and lateral acceleration rate, ft.

\( V \) = Velocity, ft/sec
\( a_y \) = Lateral acceleration rate, ft/sec²
8. \( r = S^2 / \left(14.97 f_y\right) \)  
Turning radius in a lane change procedure knowing initial speed and lateral acceleration factor, \( \text{ft} \).

\[ S = \text{Speed, mi/hr} \]
\[ f_y = \text{Lateral acceleration factor, decimal} \]

9. \( r = 0.25 \left(d_x^2 / d_y + d_y\right) \)  
Turning radius in a lane change procedure knowing the linear and lateral components of the turn, \( \text{ft} \).

\[ d_x = \text{Linear distance, ft} \]
\[ d_y = \text{Lateral turning distance, ft} \]

10. \( r = 0.067 S^2 / f_y \)  
Turning radius traveled in a lane change knowing the speed and lateral acceleration factor, \( \text{ft} \).

\[ S = \text{Speed, mi/hr} \]
\[ f_y = \text{Lateral acceleration factor, decimal} \]

11. \( f_y = d_y 0.268 / (d_x / S)^2 \)  
Lateral acceleration factor in a lane change maneuver knowing the linear and lateral distances traversed and speed, \text{decimal}.

\[ S = \text{Speed, mi/hr} \]
\[ d_x = \text{Linear distance, ft} \]
\[ d_y = \text{Lateral turning distance, ft} \]
12. \( f_y = \frac{S^2}{14.97r} \)  
   Lateral acceleration factor of a vehicle in a lane change maneuver at a constant speed with a known radius, decimal.

   \( S = \) Speed constant, mi/hr  
   \( r = \) Radius traveled by center of mass, ft

13. \( a_y = \frac{V^2}{r} \)  
   Lateral acceleration rate of a vehicle in a lane change maneuver at a constant velocity with a known radius, ft/sec\(^2\).

   \( V = \) Velocity constant, ft/sec  
   \( r = \) Radius traveled by center of mass, ft

14. \( S = d_i \frac{1.93}{\sqrt{d_y / f_y}} \)  
   Speed in a lane change maneuver knowing the lateral distance, lateral acceleration factor and distance in the turn, mi/hr.

   \( f_y = \) Lateral acceleration factor, decimal  
   \( d_y = \) Lateral turning distance, ft  
   \( d_i = \) Distance in turn, ft (Eq #4)

15. \( T = \frac{d_i}{V} \)  
   Total time in a lane change, sec.

   \( d_i = \) Distance in turn, ft (Eq #4, 5)  
   \( V = \) Velocity, ft/sec
Low Speed Impact

The following equations appear in Low Speed; rear end, lateral and sideswipe collisions April 21-25, 97 IPTM.

\[
N. \quad \Delta V_i = \frac{1}{1 + \frac{m_t}{m_b}} \sqrt{\frac{2(E_i + E_b)(m_t + m_b)}{m_t m_b}}
\]

****this is high speed, however it appears in the low speed information given by MEA

\[
E_i = \text{Energy for target vehicle, ft-lb}
\]

\[
E_b = \text{Energy for bullet vehicle, ft-lb}
\]

\[
m_t = \text{Mass of target vehicle,}
\]

\[
m_b = \text{Mass of bullet vehicle,}
\]

\[
g. \quad \Delta V_i = \frac{(1 + e)\sqrt{2(E_i + E_b)(m_t + m_b)}}{1 + \frac{m_t}{m_b}} \frac{1 - e^2}{(1 - e^2)m_t m_b}
\]

\[
N. \quad \Delta V_1 = \frac{m_2(1 + e_{vv})}{m_1 + m_2} \sqrt{\frac{2(E_{B1} + E_{B2})(m_1 + m_2)}{(1 - e_{vv}^2)m_1 m_2}}
\]

\[
N. \quad \Delta V_2 = \frac{m_1}{m_2} \Delta V_1
\]

\[
N. \quad e_{vv} = \sqrt{1 + \frac{m_1(e_{B2}^2 - 1) + m_2(e_{B1}^2 - 1)}{m_1 + m_2}}
\]

\[
N. \quad V_{R,t} = \left[\frac{(1 + m_t/m_b)}{(1 + e)}\right] \Delta V_i \quad \text{Eq 4-3}
\]
Coefficient of Restitution

23. Coefficient of restitution, decimal. (Collinear impacts)
   For perfect elastic collision, $e = 1$.
   For inelastic collisions, $e < 1$.
   If vehicles lodge together after collision,
   
   $$V_4 = V_3, \quad e = 0.$$ 
   $$e = \frac{(V_3 - V_4)}{(V_1 - V_2)}$$ 

   $V_1$ = Pre-impact velocity veh #1, ft/sec
   $V_2$ = Pre-impact velocity veh #2, ft/sec
   $V_3$ = Post-impact velocity veh #1, ft/sec
   $V_4$ = Post-impact velocity veh #2, ft/sec

G. Peak bumper force proportional to the change in speed of a stationary target vehicle, mi/hr.

$$\Delta V = \left(\frac{\mu W}{12R_0W}\right)V_L$$

$\mu$ = Roadway friction coefficient, decimal
$V_L$ = Bumper Limit Speed, mph

Watts

$R_0$ = Ratio of Energy, ft-lb
(typically 0.5 - 1.0)
$W$ = Weight of vehicle, lb

G. Closing velocity of two vehicles during a collinear impact, ft/sec.

$$V_c = \sqrt{\frac{2E_d g W_1 + W_2}{W_1 W_2 \left(1 - e^2\right)}}$$

$E_d$ = Total absorbed energy for damage from both vehicles,
ft-lb

Wells, Atkinson, Hennessy

$W_1$ = Weight of vehicle #1, lb
$W_2$ = Weight of vehicle #2, lb
$e$ = Coefficient of restitution, decimal
$g$ = Gravitational constant, 32.2 ft/sec$^2$
N. Post impact speed of Veh #2 during a collinear collision with vehicle #2 stationary prior to impact, \( \text{ft/sec} \).

\[
V_4 = \left[ \frac{W_1}{(W_1 + W_2)} \right] (1 - e) V_1
\]

\( W_1 = \text{Weight of vehicle #1, lb} \)
\( W_2 = \text{Weight of vehicle #2, lb} \)
\( e = \text{Coefficient of restitution, decimal} \)
\( V_1 = \text{Pre-impact speed of Veh #1, ft/sec} \)

N. Post impact speed of Veh #1 during a collinear collision with vehicle #2 stationary prior to impact, \( \text{ft/sec} \).

\[
V_3 = \left[ \frac{W_1 + eW_2}{(W_1 + W_2)} \right] V_1
\]

\( W_1 = \text{Weight of vehicle #1, lb} \)
\( W_2 = \text{Weight of vehicle #2, lb} \)
\( e = \text{Coefficient of restitution, decimal} \)
\( V_1 = \text{Pre-impact speed of Veh #1, ft/sec} \)

N. Delta V for the target vehicle during a collinear impact, \( \text{ft/sec} \).

\[
\Delta V_T = \frac{\sqrt{2E_D g W_2 (1 - e)^2}}{W_1 (W_1 + W_2)(1 - e^2)}
\]

\( E_D = \text{Total absorbed energy for damage from both vehicles, ft-lb} \)
\( W_1 = \text{Weight of vehicle #1, lb} \)
\( W_2 = \text{Weight of vehicle #2, lb} \)
\( e = \text{Coefficient of restitution, decimal} \)
\( g = \text{Gravitational constant, 32.2 ft/sec}^2 \)
N. Delta V for the bullet vehicle during a collinear impact, \( \text{ft/sec} \).

\[
\Delta V_b = \sqrt{\frac{2 E_D g W_1 (1 - e^2)}{W_2 (W_1 + W_2) (1 - e^2)}}
\]

- \( E_D \) = Total absorbed energy for damage from both vehicles, ft-lb
- \( W_1 \) = Weight of vehicle #1, lb
- \( W_2 \) = Weight of vehicle #2, lb
- \( e \) = Coefficient of restitution, decimal
- \( g \) = Gravitational constant, 32.2 ft/sec\(^2\)

**Mass**

The measure of the quantity of matter that a body or an object contains. The mass of the body is not dependent on gravity and therefore is different from but proportional to its weight.

1. Mass of an object relating to the weight and acceleration of gravity, \( \text{lb-sec}^2/\text{ft} \).

\[
m = \frac{W}{g}
\]

- \( W \) = Total static weight, lb
- \( g \) = Gravitational constant, 32.2 ft/sec\(^2\)

2. Mass of an object relating to the weight and acceleration of gravity, \( \text{lb-sec}^2/\text{in} \).

\[
m = \frac{W}{g}
\]

- \( W \) = Total static weight, lb
- \( g \) = Gravitational constant, 386.4 in/sec\(^2\)

**Relating to Force**

3. Mass of an object relating to the force and acceleration rate, \( \text{lb-sec}^2/\text{ft} \).

\[
m = \frac{F}{a}
\]

- \( F \) = Force, lb
- \( a \) = Acceleration rate, ft/sec\(^2\)
**Effective Mass Coefficient**

4. Effective mass coefficient at the center of gravity, decimal.

\[ \gamma = \frac{k^2}{k^2 + F_a^2} \]

\[ k^2 = \text{Radius of gyration, } \text{in}^2 \]

\[ F_a = \text{Moment arm of force, } \text{in} \]

(Eq #25, 26, 27 Damage (Crush) section)

**Inertia, Mass Moment of**

5. Mass moment of inertia, \( \text{ft-lb-sec}^2 \).

\[ I = 0.004WL\ell \]

\( W = \text{Total static weight, lb} \)
\( L = \text{Overall length of vehicle, ft} \)
\( \ell = \text{Wheelbase, ft} \)

Limpert

6. Mass moment of inertia, \( \text{in-lb-sec}^2 \).

\[ I = k^2m \]

\[ k^2 = \text{Radius of gyration, } \text{in}^2 \]

(Eq #25, 26, 27 Damage (Crush) section)
\[ m = \text{Mass, lb-sec}^2/\text{in} \] (Eq #2)

**Momentum Check**

5. \[ \rho_1 = Tan^{-1}\left[ \frac{V_3\sin\theta}{V_1 - V_3\cos\theta} \right] \]

Principal direction of force (impulse vector) for vehicle #1, \( \text{deg} \). Measured from the vehicle’s x-axis heading to the right or left quadrant.

\[ V_1 = \text{Pre-impact velocity veh #1, ft/sec} \] (Eq #2a)
\[ V_3 = \text{Post-impact velocity veh #1, ft/sec} \] (Eq #2c)
\[ \theta = \text{Departure angle vehicle #1, deg} \]
6. \( \rho_1 = \psi_1 - \alpha + 180 \)

Principal direction of force (impulse vector) for vehicle #1, deg.
RH coordinate system with a vertical x-axis. Resultant measured from the vehicle's x-axis heading to the right or left quadrant.

\( \psi_1 \) = Force angle from the global x-axis for Vehicle #1, degrees (EQ #)
\( \alpha \) = Approach angle, vehicle #1, deg

7. \( \rho_2 = \tan^{-1} \left( \frac{V_4 \cos \phi}{V_2 - V_4 \sin \phi} \right) \)

Principal direction of force (impulse vector) for vehicle #2, deg. Measured from the vehicle's x-axis heading to the right or left quadrant.

\( V_2 \) = Pre-impact velocity veh #2, ft/sec (Eq #2b)
\( V_4 \) = Post-impact velocity veh #2, ft/sec (Eq #2d)
\( \phi \) = Departure angle, vehicle #2, deg

8. \( \rho_2 = \psi_2 - \psi + 180 \)

Principal direction of force (impulse vector) for vehicle #2, deg.
RH coordinate system with a vertical x-axis. Resultant measured from the vehicle's x-axis heading to the right or left quadrant.

\( \psi_2 \) = Force angle from the global x-axis for Vehicle #2, degrees (EQ #)
\( \psi \) = Approach angle, vehicle #2, deg
Verification

9. \[ \alpha = \psi + \rho_2 - 180 - \rho_1 \]
   Verification of the approach angle for vehicle \#1 & 2 and the principal direction of forces, deg.

10. \[ \psi = \alpha + \rho_1 - \rho_2 + 180 \]

11. \[ \rho_1 = \psi + \rho_2 - 180 - \alpha \]
   \( \alpha \) = Approach angle, vehicle \#1, deg
   \( \psi \) = Approach angle, vehicle \#2, deg
   \( \rho_1 \) = Principal direction of force for vehicle \#1, deg (Eq #14)
   \( \rho_2 \) = Principal direction of force for vehicle \#2, deg (Eq #15)

12. \[ \rho_2 = \alpha + \rho_1 + 180 - \psi \]

Momentum

A measure of the motion of a body equal to the product of its mass and velocity. Also called linear momentum.

In-Line Momentum

1. \[ W_1V_1 + W_2V_2 = W_1V_3 + W_2V_4 \]
   Basic linear (in-line) momentum equation, lb-ft/sec. Less than 10° approach angle between vehicles.

   \( W_1 \) = Weight, vehicle \#1, lb
   \( W_2 \) = Weight, vehicle \#2, lb
   \( V_1 \) = Pre-impact velocity veh \#1, ft/sec
   \( V_2 \) = Pre-impact velocity veh \#2, ft/sec
   \( V_3 \) = Post-impact velocity veh \#1, ft/sec
   \( V_4 \) = Post-impact velocity veh \#2, ft/sec

   1a. \[ V_1 = \frac{W_1V_3 + W_2V_4 - W_2V_2}{W_1} \]
   1b. \[ V_2 = \frac{W_1V_3 + W_2V_4 - W_1V_1}{W_2} \]

   1c. \[ V_3 = \frac{W_1V_1 + W_2V_2 - W_2V_4}{W_1} \]
   1d. \[ V_4 = \frac{W_1V_1 + W_2V_2 - W_1V_3}{W_2} \]
Angular Momentum

Basic angular momentum equations along the x & y-axis, lb-ft/sec.

\[ W_1 = \text{Weight, vehicle #1, lb} \]
\[ W_2 = \text{Weight, vehicle #2, lb} \]
\[ V_1 = \text{Pre-impact velocity veh #1, ft/sec} \]
\[ V_2 = \text{Pre-impact velocity veh #2, ft/sec} \]
\[ V_3 = \text{Post-impact velocity veh #1, ft/sec} \]
\[ V_4 = \text{Post-impact velocity veh #2, ft/sec} \]
\[ \alpha = \text{Approach angle, vehicle #1, deg} \]
\[ \psi = \text{Approach angle, vehicle #2, deg} \]
\[ \theta = \text{Departure angle, vehicle #1, deg} \]
\[ \phi = \text{Departure angle, vehicle #2, deg} \]

2. **y axis** = \[ W_1 V_1 \sin \alpha + W_2 V_2 \sin \psi = W_1 V_3 \sin \theta + W_2 V_4 \sin \phi \]

2a. \[ V_2 = \frac{W_1 V_3 \sin \theta + W_2 V_4 \sin \phi}{W_2 \sin \psi} \]

2b. \[ V_3 = \frac{W_2 V_2 \sin \psi + W_2 V_4 \sin \phi}{W_1 \sin \theta} \]

2c. \[ V_4 = \frac{W_2 V_2 \sin \psi - W_1 V_3 \sin \theta}{W_2 \sin \phi} \]
3. \( \mathbf{x \text{ axis}} = W_1V_1\cos\alpha + W_2V_2\cos\psi = W_3V_3\cos\theta + W_4V_4\cos\phi \)

3a. \( V_1 = \frac{W_1V_3\cos\theta + W_4V_4\cos\phi - W_2V_2\cos\psi}{W_1\cos\alpha} \)

3b. \( V_2 = \frac{W_1V_3\cos\theta + W_4V_4\cos\phi - W_1V_1\cos\alpha}{W_2\cos\psi} \)

3c. \( V_3 = \frac{W_1V_4\cos\alpha + W_2V_2\cos\psi - W_4V_4\cos\phi}{W_1\cos\theta} \)

3d. \( V_4 = \frac{W_1V_4\cos\alpha + W_2V_2\cos\psi - W_3V_3\cos\theta}{W_2\cos\phi} \)

4a. \( V_1 = \frac{W_1V_3[\cos\theta\sin\psi - \sin\theta\cos\psi] + W_4V_4[\cos\phi\sin\psi - \sin\phi\cos\psi]}{W_1[\cos\alpha\sin\psi - \sin\alpha\cos\psi]} \)

4b. \( V_2 = \frac{W_1V_3[\cos\theta\sin\alpha - \sin\theta\cos\alpha] + W_4V_4[\cos\phi\sin\alpha - \sin\phi\cos\alpha]}{W_2[\cos\psi\sin\alpha - \sin\psi\cos\alpha]} \)

\textit{Numerous Departuring Structures}

N. \( V_1 = \frac{W_3V_3\cos\theta + W_4V_4\cos\phi + W_nV_n\cos_n + \cdots - W_2V_2\cos\psi}{W_1\cos\alpha} \)

N. \( V_2 = \frac{W_3V_3\sin\theta + W_4V_4\sin\phi + W_nV_n\sin_n + \cdots}{W_2\sin\psi} \)
Principal Direction of Force

\[ N. \quad \rho_1 = \sin^{-1} \left( \frac{V_3 \sin \theta}{\Delta V_1} \right) \]

Principal direction of force (impulse vector) for vehicle #1, deg. Measured from the vehicle's x-axis heading to the right or left quadrant.

\[ V_3 = \text{Post-impact velocity veh #1, ft/sec} \]  
(Eq #2c)

\[ \Delta V_1 = \text{Delta V for vehicle #1, ft/sec} \]  
(Eq #3, 4)

\[ \theta = \text{Departure angle vehicle #1, deg} \]

\[ N. \quad \rho_2 = \sin^{-1} \left( \frac{V_4 \sin (\psi - \phi)}{\Delta V_2} \right) \]

Principal direction of force (impulse vector) for vehicle #2, deg. Measured from the vehicle's x-axis heading to the right or left quadrant.

\[ V_4 = \text{Post-impact velocity veh #2, ft/sec} \]  
(Eq #2c)

\[ \Delta V_2 = \text{Delta V for vehicle #2, ft/sec} \]  
(Eq #3, 4)

\[ \psi = \text{Approach angle, vehicle #2, deg} \]
\[ \phi = \text{Departure angle vehicle #2, deg} \]

\[ N. \quad \psi \cong 180 - (\rho_1 + \rho_2) \]

John Daily's Fundamentals of Applied Physics for the accident reconstructionist p.284
\Change 13. \( \Delta V_1 = \sqrt{(V_1^2 + V_3^2)} - 2V_1V_3\cos \theta \)  
\( \Delta V_1 \) for vehicle \#1, \textbf{ft/sec}.

\( V_1 = \) Pre-impact velocity veh \#1, \text{ft/sec}  
(Eq #2a)
\( V_3 = \) Post-impact velocity veh \#1, \text{ft/sec}  
(Eq #2c)
\( \theta = \) Departure angle vehicle \#1, \text{deg}  
(Eq #22)

\Change 14. \( \Delta V_2 = \sqrt{(V_2^2 + V_4^2)} - 2V_2V_4\cos(\psi - \phi) \)  
\( \Delta V_2 \) for vehicle \#2, \textbf{ft/sec}.

\( V_2 = \) Pre-impact velocity veh \#2, \text{ft/sec}  
(Eq #2b)
\( V_4 = \) Post-impact velocity veh \#2, \text{ft/sec}  
(Eq #2d)
\( \psi = \) Approach angle, vehicle \#2, \text{deg}  
\( \phi = \) Departure angle, vehicle \#2, \text{deg}  
(Eq #22)

\Change 15. \( \Delta V_2 = (W_1 / W_2)\Delta V_1 \)  
\( \Delta V_2 \) for vehicle \#2, \textbf{ft/sec}.

\( \Delta V_1 = \) Delta V for vehicle \#1, \text{ft/sec}  
(Eq #3, 4)
\( W_1 = \) Weight, vehicle \#1, \text{lb}  
\( W_2 = \) Weight, vehicle \#2, \text{lb}
**Delta V Angle**

16. \[ \psi_1 = \cos^{-1}\left(\frac{V_3 \cos \theta - V_1 \cos \alpha}{\Delta V_1}\right) \]

Force angle for delta \( V_1 \) from the global \( x \)-axis, **degree**.

- \( \alpha \) = Approach angle, vehicle #1, deg
- \( \theta \) = Departure angle vehicle #1, deg
- \( V_1 \) = Pre-impact velocity veh #1, ft/sec
- \( V_3 \) = Post-impact velocity veh #1, ft/sec
- \( \Delta V_1 \) = Delta V for vehicle #1, ft/sec
  (Eq #3, 4)

17. \( \psi_2 = \psi_1 + 180 \)

Force angle for delta \( V_2 \) from the global \( x \)-axis, **degree**.

- \( \psi_1 \) = Force angle from the global \( x \)-axis for Vehicle #1, degrees
  (EQ #)

**Delta M**

18. \[ \Delta M_1 = \sqrt{\left(\frac{M_1^2}{M_3^2}\right) - 2M_1M_3 \cos \theta} \]

Delta M for vehicle #1, **lb-ft/sec**.

- \( M_1 \) = Pre-impact momentum veh #1, lb-ft/sec (Eq #18)
- \( M_3 \) = Post-impact momentum veh #1, lb-ft/sec (Eq #18)
- \( \theta \) = Departure angle vehicle #1, deg
  (Eq #22)
19. \[ \Delta M_2 = \sqrt{(M_2^2 + M_4^2) - 2M_2M_4\cos(\psi - \phi)} \] Delta M for vehicle #2, lb-ft/sec.

\[ M_2 = \text{Pre-impact momentum veh #2, lb-ft/sec (Eq #18)} \]
\[ M_4 = \text{Post-impact momentum veh #2, lb-ft/sec (Eq #18)} \]
\[ \psi = \text{Approach angle, vehicle #2, deg (Eq #22)} \]
\[ \phi = \text{Departure angle, vehicle #2, deg (Eq #22)} \]

20. \[ \Delta E = W\Delta V^2 / 2g \] Change in energy for either vehicle during a collision, ft-lb.

\[ W = \text{Weight of vehicle, lb} \]
\[ \Delta V = \text{Delta V. Magnitude of the velocity change for the center of gravity, ft/sec (Eq #3, 4)} \]
\[ g = \text{Gravitational constant, 32.2 ft/sec}^2 \]

**Combined energy**

21. \[ E_c = \left(\frac{m_1}{2}\right)(V_1^2 - V_3^2) + \left(\frac{m_2}{2}\right)(V_2^2 - V_4^2) \] Combined energy generated during impact, ft-lb.

\[ m_1 = \text{Mass, vehicle #1, lb-sec}^2 / \text{ft} \]
\[ m_2 = \text{Mass, vehicle #2, lb-sec}^2 / \text{ft} \]
\[ V_1 = \text{Pre-impact velocity veh #1, ft/sec} \]
\[ V_2 = \text{Pre-impact velocity veh #2, ft/sec} \]
\[ V_3 = \text{Post-impact velocity veh #1, ft/sec} \]
\[ V_4 = \text{Post-impact velocity veh #2, ft/sec} \]
22. \[ Ec = 0.0334\left[W_1\left(S_1^2 - S_3^2\right) + W_2\left(S_2^2 - S_4^2\right)\right] \] Combined energy generated during impact, \textbf{ft-lb}.

\[ W_1 = \text{Weight, vehicle #1, lb} \]
\[ W_2 = \text{Weight, vehicle #2, lb} \]
\[ S_1 = \text{Pre-impact speed veh #1, mi/hr} \]
\[ S_2 = \text{Pre-impact speed veh #2, mi/hr} \]
\[ S_3 = \text{Post-impact speed veh #1, mi/hr} \]
\[ S_4 = \text{Post-impact speed veh #2, mi/hr} \]

24. \[ M = SW \] Momentum of an object due to speed and weight, \textbf{lb-mi/hr}.

\[ S = \text{Speed constant, mi/hr} \]
\[ W = \text{Weight, lb} \]

25. \[ M = VW \] Momentum of an object due to velocity and weight, \textbf{lb-ft/sec}.

\[ V = \text{Velocity, ft/sec} \]
\[ W = \text{Weight, lb} \]

26. \[ M = Vm \] Momentum of an object due to velocity and mass, \textbf{lb-sec}.

\[ V = \text{Velocity, ft/sec} \]
\[ m = \text{Mass, lb-sec}^2/\text{ft} \]

27. \[ M = \frac{2Fd}{V} \] Momentum of an object due to velocity, force applied and distance, \textbf{lb-sec}.

\[ V = \text{Velocity constant, ft/sec} \]
\[ F = \text{Force, lb} \]
\[ d = \text{Distance, f} \]
28. \( d_t = \sqrt{d_x^2 + d_y^2} \) 

Vehicle departure distance commencing from point of impact to point of rest, \textbf{ft}.

Pythagorean's theorem.

\( d_x = \) Vehicle's linear distance from point of impact to a point perpendicular to point of rest, \textbf{ft}

\( d_y = \) Vehicle's lateral distance from point of impact to a point perpendicular to point of rest, \textbf{ft}

29. \( \theta = \sin^{-1}\left(\frac{d_y}{d_t}\right) \) 

Vehicle departure angle commencing from point of impact to point of rest, \textbf{deg}.

\( d_y = \) Vehicle's lateral distance from point of impact to a point perpendicular to point of rest, \textbf{ft}

\( d_t = \) Vehicle's departure distance, \textbf{ft}  

(Eq #21)

31. Coefficient of restitution, \textbf{decimal}. (Collinear impacts)

\( e = \frac{(V_3 - V_4)}{(V_1 - V_2)} \)

\( V_1 = \) Pre-impact velocity veh #1, \textbf{ft/sec m/sec}

\( V_2 = \) Pre-impact velocity veh #2, \textbf{ft/sec m/sec}

\( V_3 = \) Post-impact velocity veh #1, \textbf{ft/sec m/sec}

\( V_4 = \) Post-impact velocity veh #2, \textbf{ft/sec m/sec}

For perfect elastic collision, \( e = 1. \)

For inelastic collisions \( e < 1. \)

If vehicles lodge together after collision, \( V_4 = V_3, \) \( e = 0. \)
Motorcycle Impact

1. Equivalent deceleration factor for a two axle vehicle during a straight line skid, knowing the Center of mass location ($x_{Fi}$ and $z_i$). Center of mass utilized as a decimal fraction of the wheelbase, **decimal**.

$$f_e = \frac{f_F - x_{Fi}(f_F - f_R)}{1 - z_i(f_F - f_R)}$$

- $f_F$ = Front deceleration factor, decimal
- $f_R$ = Rear deceleration factor, decimal
- $x_{Fi}$ = Longitudinal center of mass from the front axle, decimal (Eq #2)
- $z_i$ = Vertical center of mass height, decimal (Eq #4)

2. Longitudinal center of mass location from the front axle as a decimal fraction of the wheelbase, **decimal**.

$$x_{Fi} = \frac{W_R}{W}$$

- $W_R$ = Static rear axle weight, lb
- $W = $ Total static weight, lb

3. Longitudinal center of mass location from the rear axle as a decimal fraction of the wheelbase, **decimal**.

$$x_{Ri} = \frac{W_F}{W}$$

- $W_F$ = Static front axle weight, lb
- $W = $ Total static weight, lb
4. Vertical center of mass height as a decimal fraction of the wheelbase, decimal.

\[ z_i = \frac{\sqrt{\ell^2 - h^2 (W_h - W_R)} + r / \ell}{Wh} \]

\( \ell = \) Wheelbase, ft
\( h = \) Height front axle elevated, ft
(3.3 feet recommended)
\( r = \) Radius of the drive wheel, ft
\( W_h = \) Rear axle weight, front elevated, lb
\( W_R = \) Static rear axle weight, lb
\( W = \) Total static weight, lb

5. Initial speed of a deceleration to a stop on a surface grade less than 11.9 %, mi/hr.

\[ S = \sqrt{30d(\mu n \pm m)} \]

\( d = \) Distance, ft
\( \mu = \) Level friction coefficient, decimal
\( m = \) Grade, maximum 11.9\%, decimal
\( n = \) Braking efficiency, decimal

6. Initial velocity of a deceleration to a stop on a surface grade less than 6.8° (11.9 %), ft/sec.

\[ V = \sqrt{2gd(\mu n \pm m)} \]

\( d = \) Distance, ft
\( \mu = \) Level friction coefficient, decimal
\( g = \) Gravitational constant, 32.2 ft/sec²
\( m = \) Grade, maximum 11.9\%, decimal
\( n = \) Braking efficiency, decimal
7. Speed of a motorcycle at point of impact when the rider's angle of take-off exceeds 6.8°, \text{mi/hr}.

\[ S = \frac{2.73d}{\cos \theta \sqrt{d \tan \theta - h}} \]

- \( d \) = Distance rider thrown from point of impact to first contact, \text{ft}
- \( h \) = Vertical distance, \text{ft} (negative value (-) for a lower center of mass landing)
- \( \theta \) = Rider launch angle, \text{deg} (Table #2)

8. Velocity of a motorcycle at point of impact when the rider's angle of take-off exceeds 6.8°, \text{ft/sec}.

\[ V = \frac{4.01d}{\sqrt{d \cos \theta \sin \theta - h \cos^2 \theta}} \]

- \( d \) = Distance rider thrown from point of impact to first contact, \text{ft}
- \( h \) = Vertical distance, \text{ft} (negative value (-) for a lower center of mass landing)
- \( \theta \) = Rider launch angle, \text{deg} (Table #2)

9. Velocity of a motorcycle at point of impact when the rider's angle of take-off exceeds 6.8°, \text{ft/sec}.

\[ V = \sqrt{\frac{2fgd}{(\cos \theta + f \sin \theta)^2}} \]

- \( f \) = Deceleration factor of rider, decimal \text{Searle}
  - 0.66 asphalt - 0.79 grass
- \( SAE 831622 \)
- \( d \) = Distance rider thrown from point of impact to point of rest, \text{ft}
- \( \theta \) = Rider launch angle, \text{deg} (Table #2)
- \( g \) = Gravitational constant, \text{32.2 ft/sec}^2
10. Minimum velocity of a motorcycle at point of impact with rider airborne, **ft/sec**.

\[ V_{\text{min}} = \sqrt{2.0gd / (1 + f^2)} \]

\( f \) = Deceleration factor of rider, decimal
\( Searle \) = asphalt - 0.79 grass
\( SAE\ 831622 \)
\( d \) = Horizontal throw distance of rider from point of impact to point of rest, ft
\( g \) = Gravitational constant, 32.2 ft/sec\(^2\)

11. Maximum velocity of a motorcycle at point of impact with rider airborne, **ft/sec**.

\[ V_{\text{max}} = \sqrt{2fgd} \]

\( f \) = Deceleration factor of rider, decimal
\( Searle \) = asphalt - 0.79 grass
\( SAE\ 831622 \)
\( d \) = Horizontal throw distance of rider from point of impact to point of rest, ft
\( g \) = Gravitational constant, 32.2 ft/sec\(^2\)

12. Distance the rider will travel from point of impact to point of rest, **ft**.

\[ d = \frac{S\sqrt{z}}{2.73} + \frac{S^2}{24} \]

\( S \) = Speed of motorcycle at impact, mi/hr
\( Collins \)
\( z \) = Vertical center of mass height of rider prior to departure, ft

**Lean Angle**

13. Maximum lean angle at which a motorcycle will negotiate a curve without loosing stability, **deg**.
(32° - 34° maximum)

\[ \alpha = \tan^{-1}(S^2 / (15r)) \]

\( S \) = Speed, mi/hr
\( r \) = Radius of curve, ft
14. Lean angle of a motorcycle negotiating a curve, deg. Measured from vertical upright position.

\[ \alpha = \tan^{-1} \left( \frac{V^2}{rg} \right) \]

\( V = \) Velocity, ft/sec
\( r = \) Radius of curve, ft
\( g = \) Gravitational constant, 32.2 ft/sec\(^2\)

15. Maximum lean angle possible for a motorcycle to maintain a curve prior to side sliding, deg. (32° - 34° maximum)

\[ \alpha = \tan^{-1} \mu \]

\( \mu = \) Friction coefficient, decimal

16. Velocity of a motorcycle in a turn knowing the lean angle and radius traveled, ft/sec.

\[ V = \sqrt{rg \tan \alpha} \]

\( r = \) Radius traveled, ft
\( \alpha = \) Lean angle, deg
  (Measured from vertical upright position)
\( g = \) Gravitational constant, 32.2 ft/sec\(^2\)

**Radius Traveled**

17. Radius traveled while negotiating a curve based on a known velocity and lean angle, ft.

\[ r = \frac{V^2}{g \tan \alpha} \]

\( V = \) Velocity, ft/sec
\( \alpha = \) Lean angle, deg
  (Measured from vertical upright position)
\( g = \) Gravitational constant, 32.2 ft/sec\(^2\)

18. Radius traveled while negotiating a curve based on a known velocity and lateral acceleration factor, ft.

\[ r = \frac{V^2}{f_y g} \]

\( V = \) Velocity, ft/sec
\( f_y = \) Lateral acceleration factor, decimal
\( g = \) Gravitational constant, 32.2 ft/sec\(^2\)
19. Radius traveled while negotiating a curve based on a known velocity and lateral acceleration rate, **ft**.

\[ r = \frac{V^2}{a_y} \]

- **V** = Velocity, **ft/sec**
- **a_y** = Lateral acceleration rate, **ft/sec^2**

20. Radius traveled while negotiating a curve based on a known speed and lateral acceleration factor, **ft**.

\[ r = \frac{S^2}{15 f_y} \]

- **S** = Speed, **mi/hr**
- **f_y** = Lateral acceleration factor, **decimal**

21. Speed of a motorcycle in a turn knowing the radius and lateral acceleration factor, **mi/hr**.

\[ S = 3.86 \sqrt{f_y r} \]

- **r** = Radius traveled, **ft**
- **f_y** = Lateral acceleration factor, **decimal**

22. Velocity of a motorcycle in a turn knowing the radius and lateral acceleration factor, **ft/sec**.

\[ V = \sqrt{f_y r g} \]

- **r** = Radius traveled, **ft**
- **f_y** = Lateral acceleration factor, **decimal**
- **g** = Gravitational constant, 32.2 **ft/sec^2**

**Lateral Acceleration**

23. Lateral acceleration factor needed to maintain the radius of a level curve at a determined speed, **decimal**.

\[ f_y = \frac{S^2}{15r} \]

- **S** = Speed, **mi/hr**
- **r** = Radius of roadway, **ft**
24. Lateral acceleration factor needed to maintain the radius of a level curve at a determined velocity, decimal.

\[ f_y = \frac{V^2}{rg} \]

\(V\) = Velocity, ft/sec
\(r\) = Radius of roadway, ft
\(g\) = Gravitational constant, 32.2 ft/sec^2

**Impact Speed, Crush**

25. Impact speed due to wheelbase reduction, mi/hr. Use only with spoke type wheels to a maximum wheelbase reduction of 13 inches. Deformation must be parallel to the motorcycle. Do not apply to customized front forks.

\[ S = 2.18\ell_r + 10.3 \]

\(\ell_r\) = Wheelbase reduction, in

Severy
SAE 700897

26. Impact speed from vehicle crush depth inflicted by the motorcycle, mi/hr.

\[ S = \frac{617Cr + 6675}{W} \]

\(Cr\) = Maximum Crush depth of vehicle, in
\(W\) = Total static weight of motorcycle, lb

27. Speed of a motorcycle from transmission measurements, mi/hr.

\[ S = \frac{\omega_e r}{i \times 168} \]

\(\omega_e\) = Rotational speed of the engine, rev/min
\(r\) = Radius of rear wheel, in
\(i\) = Final drive-gear ratio, 00:1 (Eq #31)

28. Determine the rotational speed of a motorcycle’s engine from transmission measurements, rev/min.

\[ \omega_e = \frac{S \times i \times 168}{r} \]

\(S\) = Speed, mi/hr
\(r\) = Radius of rear wheel, in
\(i\) = Final drive-gear ratio, 00:1 (Eq #31)
29. Radius of the drive wheel from transmission measurements, \textit{in.}

\[ r = \frac{S \times i \times 168}{\omega_e} \]

\( \omega_e = \) Rotational speed of the engine, rev/min

\( S = \) Speed, mi/hr

\( i = \) Final drive-gear ratio, 00:1 (Eq #31)

30. Final drive-gear ratio of a motorcycle, \textbf{00:1}.

\[ i = \frac{\omega_e r}{168S} \]

\( \omega_e = \) Rotational speed of the engine, rev/min

\( S = \) Speed, mi/hr

\( r = \) Radius of rear wheel, in

31. Final drive gear ratio of a motorcycle utilizing interior gear mechanisms, \textbf{00:1}.

\[ i = \left( \frac{R_{ws}}{G_{bs}} \right) \left( \frac{C_s}{E_s} \right) \]

\( R_{ws} = \) Rear wheel sprocket, \# teeth

\( G_{bs} = \) Gear box sprocket, \# teeth

\( C_s = \) Clutch sprocket, \# teeth

\( E_s = \) Engine sprocket, \# teeth

32. Velocity of a motorcycle involved in a 90 degree impact with a motor vehicle, \textbf{ft/sec}.

\[ V = \sqrt{\frac{W_V \mu d}{W_M / 64.4}} \]

\( W_V = \) Total weight of vehicle, lb

\( W_M = \) Total weight of motorcycle, lb

\( \mu = \) Friction coefficient, decimal

\( d = \) Post-impact distance traveled while sliding, ft
Newton’s Laws of Motion

Sir Isaac Newton (1642 – 1727)

**First Law:** *(Equilibrium/Inertia)* A body at rest remains at rest and when in uniform motion continues to move in uniform motion along a straight line unless, in either case, the body is acted upon by an unbalanced force.

\[ \sum F_n = 0 \]

**Second Law:** *(Impulse)* An unbalanced force acting on a body causes the body to accelerate in the direction of the line of action of force; the acceleration is directly proportional to the force and inversely proportional to the mass of the body.

\[ F = ma \quad a = \frac{F}{m} \]

**Third Law:** To every action there is always opposed an equal reaction; or, the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.

\[ (F_1) = (-F_2) \]
Oblique Angle Collisions

Closing Velocity

\[ N1. \quad V_c = \sqrt{V_1^2 + V_2^2 - 2V_1V_2 \cos \theta} \]

Closing velocity in an oblique angle collision, \textbf{ft/sec}.

\[ \theta = \text{Approach angle of vehicle #2 relative to vehicle #1, degrees} \]

\[ V_1 = \text{Velocity of vehicle #1, ft/sec} \]

\[ V_2 = \text{Velocity of vehicle #2, ft/sec} \]

Closing Distance

\[ N2. \quad d_c = \sqrt{d_1^2 + d_2^2 - 2d_1d_2 \cos \theta} \]

Closing distance in an oblique angle collision, \textbf{ft}.

\[ d_1 = V_1T_1 \]

\[ d_2 = V_2T_2 \]

\[ T_1 = \text{Time to impact for vehicle #1, sec} \]

\[ T_2 = \text{Time to impact for vehicle #2, sec} \]

Off Tracking / Low Speed Turn

Effective Turn Angle, Maximum

1. Maximum effective turn angle to the front wheels knowing the degrees, \textbf{rad}.

\[ \delta = \delta_1 \pi / 180 \]

\[ \delta_1 = \text{Effective turn angle to the front wheels, deg} \]

\[ \pi = \text{Pi, 3.141592654} \]

2. Maximum effective turn angle to the front wheels knowing the radians, \textbf{deg}.

\[ \delta_1 = \delta_1 80 / \pi \]

\[ \delta = \text{Effective turn angle to the front wheels, rad} \]

\[ \pi = \text{Pi, 3.141592654} \]
3. Maximum effective turn angle to the front wheels during a turn knowing the radius and wheelbase, deg.

\[ \delta_i = \sin^{-1} \left( \frac{\ell}{r} \right) \]
\( \ell = \) Wheelbase, ft
\( r = \) Radius traveled by center of mass, ft

N. Steering angle for the outside front tires during a low speed turn, rad.

\[ \delta_o = \frac{\ell}{r + tw/2} \]
\( \ell = \) Wheelbase, ft
\( tw = \) Track width, ft

Gillespie
\( r = \) Radius traveled by center of mass, ft

N. Steering angle for the inside front tires during a low speed turn, rad.

\[ \delta_i = \frac{\ell}{r - tw/2} \]
\( \ell = \) Wheelbase, ft
\( tw = \) Track width, ft

Gillespie
\( r = \) Radius traveled by center of mass, ft

Radius

4. Radius traveled during a turn knowing the wheelbase of the vehicle and the radians of an effective turn angle, ft.

\[ r = \frac{\ell}{\delta} \]
\( \ell = \) Wheelbase, ft
\( \delta = \) Effective turn angle to the front wheels, rad (Eq #1)

5. Radius traveled during a turn knowing the wheelbase of the vehicle and the degrees of an effective turn angle, ft.

\[ r = \frac{\ell}{\sin \delta_1} \]
\( \ell = \) Wheelbase, ft
\( \delta_1 = \) Effective turn angle to the front wheels, deg (Eq #2)
**Steering Wheel, Total Rotation**

6. Total rotation to the steering wheel during a turn, \( \text{deg.} \)

\[
Sr = \delta_1 \cdot fs
\]

\( \delta_1 \) = Effective turn angle to the front wheels, \( \text{deg} \) (Eq #2)

\( fs \) = Steering ratio, 00:1

---

**Non-articulated Vehicle**

7. Turning radius of the rear axle on a non-articulated vehicle during a low speed turn, \( \text{ft.} \)

\[
r_R = \sqrt{r_F^2 - \ell^2}
\]

\( r_F \) = Turning radius of front axle (center), \( \text{ft} \)

\( \ell \) = Wheelbase, \( \text{ft} \)

8. Inside minimum turning radius of a non-articulated vehicle during a low speed turn, \( \text{ft.} \)

\[
r_{\min} = \ell / \delta - w / 2
\]

\( \ell \) = Wheelbase, \( \text{ft} \)

\( \delta \) = Effective turn angle to the front wheels, \( \text{rad} \) (Eq #1)

\( w \) = Vehicle width, \( \text{ft} \)

9. Maximum turning radius of the outside front corner of a non-articulated vehicle during a low speed turn, \( \text{ft.} \)

\[
r_{\max} = \sqrt{(\ell / \delta + w / 2)^2 + x_b^2}
\]

\( \ell \) = Wheelbase, \( \text{ft} \)

\( \delta \) = Effective turn angle to the front wheels, \( \text{rad} \) (Eq #1)

\( w \) = Vehicle width, \( \text{ft} \)

\( x_b \) = Front bumper to rear axle distance, \( \text{ft} \)
**Articulated Vehicle**

10. Maximum off tracking on a tractor/semi trailer during a low speed turn, \( \text{ft} \).

\[
Ot = r_F - \sqrt{r_F^2 + \ell_k^2 - \ell_1^2 - \ell_2^2}
\]

\( r_F \) = Turning radius of tractor's front axle (center), \( \text{ft} \)

\( \ell_k \) = Distance forward from the tractor's rear axle to the 5th wheel kingpin, \( \text{ft} \)

\( \ell \) = Tractor wheelbase, \( \text{ft} \)

\( \ell_1 \) = Semi trailer wheelbase, \( \text{ft} \)

**Double Configuration**

11. Maximum off tracking on a double configuration during a low speed turn, \( \text{ft} \).

\[
Ot = r_F - \sqrt{r_F^2 + \ell_k^2 - \ell_1^2 - \ell_2^2 - \ell_3^2 - \ell_4^2}
\]

\( r_F \) = Turning radius of tractor's front axle (center), \( \text{ft} \)

\( \ell_k \) = Distance forward from the tractor's rear axle to the 5th wheel kingpin, \( \text{ft} \)

\( \ell \) = Tractor wheelbase, \( \text{ft} \)

\( \ell_1 \) = Semi trailer wheelbase, \( \text{ft} \)

\( \ell_2 \) = Rearward overhang of the pintle hitch, \( \text{ft} \)

\( \ell_3 \) = Dolly drawbar length, \( \text{ft} \)

\( \ell_4 \) = Full trailer wheelbase, \( \text{ft} \)

**Triple Configuration**

12. Maximum off tracking on a triple configuration during a low speed turn, \( \text{ft} \).

\[
Ot = r_F - \sqrt{r_F^2 + \ell_k^2 - \ell_1^2 - \ell_2^2 - \ell_3^2 - \ell_4^2 - \ell_5^2 - \ell_6^2 - \ell_7^2}
\]

\( r_F \) = Turning radius of tractor's front axle (center), \( \text{ft} \)

\( \ell_k \) = Distance forward from the tractor's rear axle to the 5th wheel kingpin, \( \text{ft} \)

\( \ell \) = Tractor wheelbase, \( \text{ft} \)

\( \ell_1 \) = Semi trailer wheelbase, \( \text{ft} \)
\( \ell_2 \) = Rearward overhang of the first pintle hitch, \( \text{ft} \)
\( \ell_3 \) = First dolly drawbar length, \( \text{ft} \)
\( \ell_4 \) = First full trailer wheelbase, \( \text{ft} \)
\( \ell_5 \) = Rearward overhang of the second pintle hitch located between the two full trailers, \( \text{ft} \)
\( \ell_6 \) = Second dolly drawbar length located between the two full trailers, \( \text{ft} \)
\( \ell_7 \) = Second full trailer wheelbase, \( \text{ft} \)

**Rear Axle**

13. Maximum off tracking of the furthest rear axle from the front axle during a low speed turn, \( \text{ft} \).

\[
Ot = r_F - r_R + \left( tw_R - tw_F \right) / 2
\]

\( r_F \) = Turning radius of front axle (center), \( \text{ft} \)
\( r_R \) = Turning radius of the furthest rear axle from the front axle (center), \( \text{ft} \) (Eq #7, 14 thru 16)
\( tw_F \) = Track width of front axle, \( \text{ft} \)
\( tw_R \) = Track width of the furthest rear axle from the front axle, \( \text{ft} \)

14. Turning radius of the trailer's rear axle on a tractor/semi trailer during a low speed turn (center), \( \text{ft} \).

\[
r_R = \sqrt{r_F^2 + \ell_k^2 - \ell_1^2}
\]

\( r_F \) = Turning radius of tractor's front axle (center), \( \text{ft} \)
\( \ell_k \) = Distance forward from the tractor's rear axle to the 5th wheel kingpin, \( \text{ft} \)
\( \ell \) = Tractor wheelbase, \( \text{ft} \)
\( \ell_1 \) = Semi trailer wheelbase, \( \text{ft} \)
15. Turning radius of the last trailer's rear axle on a double configuration during a low speed Turn (center), ft.

\[ r_R = \sqrt{r_F^2 + \ell_k^2 - \ell^2 - \ell_1^2 - \ell_2^2 - \ell_3^2 - \ell_4^2} \]

\[ r_F = \text{Turning radius of tractor's front axle (center), ft} \]
\[ \ell_k = \text{Distance forward from the tractor's rear axle to the 5th wheel kingpin, ft} \]
\[ \ell = \text{Tractor wheelbase, ft} \]
\[ \ell_1 = \text{Semi trailer wheelbase, ft} \]
\[ \ell_2 = \text{Rearward overhang of the pintle hitch, ft} \]
\[ \ell_3 = \text{Dolly drawbar length, ft} \]
\[ \ell_4 = \text{Full trailer wheelbase, ft} \]

16. Turning radius of the last trailer's rear axle on a triple configuration during a low speed Turn (center), ft.

\[ r_R = \sqrt{r_F^2 + \ell_k^2 - \ell^2 - \ell_1^2 - \ell_2^2 - \ell_3^2 - \ell_4^2 - \ell_5^2 - \ell_6^2 - \ell_7^2} \]

\[ r_F = \text{Turning radius of tractor's front axle (center), ft} \]
\[ \ell_k = \text{Distance forward from the tractor's rear axle to the 5th wheel kingpin, ft} \]
\[ \ell = \text{Tractor wheelbase, ft} \]
\[ \ell_1 = \text{Semi trailer wheelbase, ft} \]
\[ \ell_2 = \text{Rearward overhang of the first pintle hitch, ft} \]
\[ \ell_3 = \text{First dolly drawbar length, ft} \]
\[ \ell_4 = \text{First full trailer wheelbase, ft} \]
\[ \ell_5 = \text{Rearward overhang of the second pintle hitch located between the two full trailers, ft} \]
\[ \ell_6 = \text{Second dolly drawbar length located between the two full trailers, ft} \]
\[ \ell_7 = \text{Second full trailer wheelbase, ft} \]
Passing Maneuver (Constant Velocity)

1. Linear distance required for first lane change, passing phase and lane change return, ft.

\[ d = V_1 \left[ \frac{L_1 + L_2 + L_S}{V_1 - V_2} + 0.9 \sqrt{d_y / f_y} \right] \]

*Limpert*

- \( V_1 \) = Velocity of passing vehicle, ft/sec
- \( V_2 \) = Velocity of vehicle being passed, ft/sec
- \( L_1 \) = Length of passing vehicle, ft
- \( L_2 \) = Length of vehicle being passed, ft
- \( L_S \) = Total length of linear safety distance between vehicles before & after passing, ft
- \( d_y \) = Lateral distance, ft Determined by adding one-half width of both vehicles and lateral safety distance.
- \( f_y \) = Lateral acceleration factor, decimal (first lane change and return)

2. Time required for complete passing maneuver, sec.

\[ T = \frac{d}{V_1} \]

- \( V_1 \) = Velocity of passing vehicle, ft/sec
- \( d \) = Total linear distance of maneuver, ft (Eq #1)

Passing Maneuver (Acceleration)

1. Velocity of a passing vehicle after first lane change, ft/sec.

\[ V_1 = 0.45a \sqrt{d_y / f_y} + V \]

- \( V \) = Velocity of vehicles before passing maneuver commences, ft/sec
- \( f_y \) = Lateral acceleration factor of first lane change, decimal
- \( a \) = Acceleration rate of passing vehicle, ft/sec\(^2\)
- \( d_y \) = Lateral distance, ft Determined by adding one-half width of both vehicles and lateral safety distance.
2. Linear distance required for the first lane change, \( ft \).

\[
d_1 = 0.45 \sqrt{d_y / f_y (V + V_1)} / 2
\]

\( V \) = Velocity of vehicles before passing maneuver commences, \( ft/sec \)

\( V_1 \) = Velocity of the passing vehicle after first lane change, \( ft/sec \) (Eq #1)

\( f_y \) = Lateral acceleration factor of first lane change, decimal

\( d_y \) = Lateral distance, \( ft \) Determined by adding one-half width of both vehicles and lateral safety distance.

3. Linear distance required for the passing phase, \( ft \). Trial & Error. The following equations for \( d_2 \) and \( V_3 \) (Eq #3, 4) require trial and error. Apply an estimated value for \( V_3 \) in equation 3. The resulting value \( (d_2) \) in equation 3 is then utilized in equation 4. If the estimated value \( (V_3') \) and the calculated value for \( V_3 \) from equation 4 are not approximately equal, adjust the estimated value \( (V_3' \) in Eq #3) until they calculate equally.

\[
d_2 = \left( \frac{L_1 + L_2 + L_S}{V_1 + V_3} \right) \frac{V_1 + V_3}{2}
\]

\( L_1 \) = Length of passing vehicle, \( ft \)

\( L_2 \) = Length of vehicle being passed, \( ft \)

\( L_S \) = Total length of linear safety distance between vehicles before & after passing, \( ft \)

\( V \) = Velocity of vehicles before passing maneuver commences, \( ft/sec \)

\( V_1 \) = Velocity of the passing vehicle after first lane change, \( ft/sec \) (Eq #1)

\( V_3 \) = Estimated velocity of passing vehicle at completion, \( ft/sec \)
4. Passing vehicle’s velocity on phase completion, **ft/sec**. Trial & Error.

\[ V_3 = \sqrt{V_1^2 + 2ad_2} \]

- \( V_1 \) = Velocity of the passing vehicle after first lane change, ft/sec (Eq #1)
- \( a \) = Acceleration rate of passing vehicle, ft/sec²
- \( d_2 \) = Linear distance required for passing phase, ft (Eq #3)

5. Return lane change velocity of the passing vehicle, **ft/sec**.

\[ V_2 = 0.45a \sqrt{d_y / f_y} + V_3 \]

- \( V_3 \) = Velocity of the passing vehicle on phase completion, ft/sec (Eq #4)
- \( f_y \) = Lateral acceleration factor of return lane change, decimal
- \( a \) = Acceleration rate of passing vehicle, ft/sec²
- \( d_y \) = Lateral distance, ft Determined by adding one-half width of both vehicles and lateral safety distance.

6. Linear distance required for lane change return, **ft**.

\[ d_3 = 0.45 \sqrt{d_y / f_y (V_2 + V_3)} / 2 \]

- \( V_2 \) = Velocity of the passing vehicle during return lane change, ft/sec (Eq #5)
- \( V_3 \) = Velocity of the passing vehicle on phase completion, ft/sec (Eq #4)
- \( f_y \) = Lateral acceleration factor of return lane change, decimal
- \( d_y \) = Lateral distance, ft Determined by adding one-half width of both vehicles and lateral safety distance.
7. Total distance required for phase completion, ft.

\[ d_T = d_1 + d_2 + d_3 \]

\[ d_1 = \text{Linear distance of first lane change, ft} \quad (\text{Eq} \ #2) \]
\[ d_2 = \text{Linear distance required for passing phase, ft} \quad (\text{Eq} \ #3) \]
\[ d_3 = \text{Linear distance required for lane change return, ft} \quad (\text{Eq} \ #6) \]

8. First or return lane change time calculated separately, sec.

\[ T_1 = 0.45 \sqrt{d_y / f_y} \]

\[ f_y = \text{Lateral acceleration factor of first lane change, decimal} \]
\[ d_y = \text{Lateral distance, ft} \quad \text{Determined by adding one-half width of both vehicles and lateral safety distance.} \]


\[ T_2 = \frac{2d_2}{V_1 + V_3} \]

\[ V_1 = \text{Velocity of the passing vehicle during first lane change, ft/sec} \quad (\text{Eq} \ #1) \]
\[ V_3 = \text{Velocity of the passing vehicle on phase completion, ft/sec} \quad (\text{Eq} \ #4) \]
\[ d_2 = \text{Linear distance required for passing phase, ft} \quad (\text{Eq} \ #3) \]

10. Time required for complete maneuver, sec.

\[ T_3 = \frac{2d_T}{V_1 + V_3} \]

\[ V_1 = \text{Velocity of the passing vehicle during first lane change, ft/sec} \quad (\text{Eq} \ #1) \]
\[ V_3 = \text{Velocity of the passing vehicle on phase completion, ft/sec} \quad (\text{Eq} \ #4) \]
\[ d_T = \text{Total linear distance required for phase completion, ft} \quad (\text{Eq} \ #7) \]
Pedestrian Impact

**Wrap:** In the wrap trajectory, the pedestrian is struck in the lower legs by the front of a decelerating vehicle. The striking portion of the vehicle must be lower than the height of the pedestrian. Upon impact the legs buckle and the torso bends over the hood and the chest impacts the top of the hood. The head impacts the hood in a whipping motion. After initial impact, the pedestrian tends to stay on the hood of the car and rides to a stop, sometimes sliding off the hood at stop. The normal take off angle is 10-20 degrees.

**Forward Projection:** In this configuration the pedestrian is struck by a flat faced vehicle, such as a truck or van, and the force applied is well above the center of gravity of the pedestrian. This can also occur when passenger vehicles strike small children. The pedestrian is quickly accelerated to the speed of the striking vehicle and then drops to the roadway surface ahead of the vehicle. The angle for a forward projection is 0 degrees for all collision types.

**Fender Vault:** This involves pedestrians struck near a front corner of the vehicle. First contact is usually made at the legs, with the torso pivoting towards the hood. Due to the position of the pedestrian (near the vehicle’s edge) he/she falls off the edge and does not impact the hood, striking the roadway. The pedestrians head may or may not impact the vehicle.

**Roof Vault:** This begins initially like a wrap trajectory but in this case the pedestrian’s legs do not stay ahead of the vehicle. Due to the impact forces the legs continue to rotate upward, with the pedestrian essentially standing on his head on or near the roofline. The vault maneuver is completed when the pedestrian leaves the vehicle. Over the roof and tumbles to the ground.

**Somersault:** This is similar in its initiation to the roof vault. During a somersault the vehicle is typically decelerating at impact and this causes the pedestrian to be thrown ahead of the vehicle. One would expect serious or even fatal head injuries as a result of this impact type. The impact orientations discussed here are applicable primarily to adult pedestrians. They may not always be applicable to small children due to their height.

**Table 2: Pedestrian Average Walking/Running Velocities**

<table>
<thead>
<tr>
<th>Age &amp; Sex</th>
<th>Pedestrian Velocity (ft/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men &lt; 55 years</td>
<td>5.4</td>
</tr>
<tr>
<td>Men &gt; 55 years</td>
<td>5.0</td>
</tr>
<tr>
<td>Women &lt; 50 years</td>
<td>4.5</td>
</tr>
<tr>
<td>Women &gt; 50 years</td>
<td>4.3</td>
</tr>
<tr>
<td>Women with small children</td>
<td>2.3</td>
</tr>
<tr>
<td>Children 17-24 months</td>
<td></td>
</tr>
<tr>
<td>Activity</td>
<td>Value</td>
</tr>
<tr>
<td>------------------------</td>
<td>-------</td>
</tr>
<tr>
<td>Dawdling</td>
<td>1.9</td>
</tr>
<tr>
<td>Walking</td>
<td>3.3</td>
</tr>
<tr>
<td>Running</td>
<td>4.1</td>
</tr>
<tr>
<td>Children 6-10 years</td>
<td>3.7</td>
</tr>
<tr>
<td>Adolescents</td>
<td>5.9</td>
</tr>
<tr>
<td>Joggers</td>
<td>9.0+</td>
</tr>
</tbody>
</table>

Walking backwards requires 40-50% of above values for the appropriate class.

<table>
<thead>
<tr>
<th>Pedestrian Action</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pedestrian to turn quickly 180°</td>
<td>0.5s</td>
</tr>
<tr>
<td>Pedestrian to turn at a normal pace 180°</td>
<td>1.3s</td>
</tr>
<tr>
<td>Elderly to turn at a normal pace 180°</td>
<td>2-2.5s</td>
</tr>
</tbody>
</table>

*Ashton / Barzeley*

**Table 3: Pedestrian Accelerations (ft/sec²)**

<table>
<thead>
<tr>
<th>Activity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pedestrian's normal walking acceleration</td>
<td>1.6</td>
</tr>
<tr>
<td>Pedestrian in a hurry</td>
<td>3.2</td>
</tr>
</tbody>
</table>

**Table 4: Pedestrian Velocity as a Function of Approaching Vehicle Proximity**

<table>
<thead>
<tr>
<th>Vehicle; Time to Impact</th>
<th>Pedestrian Velocities (ft/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle ≤ 8 seconds away</td>
<td>4.0</td>
</tr>
<tr>
<td>Vehicle ≤ 6 seconds away</td>
<td>4.1</td>
</tr>
<tr>
<td>Vehicle ≤ 4 seconds away</td>
<td>4.7</td>
</tr>
<tr>
<td>Vehicle ≤ 2 seconds away</td>
<td>6.5</td>
</tr>
</tbody>
</table>

*Ashton*

**Table 5: Pedestrian Deceleration Factors**

<table>
<thead>
<tr>
<th>Function / Surface</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sliding</td>
<td></td>
</tr>
<tr>
<td>Grass</td>
<td>.45 - .70</td>
</tr>
<tr>
<td>Asphalt</td>
<td></td>
</tr>
<tr>
<td>Leathers</td>
<td>.60 - .70</td>
</tr>
<tr>
<td>Polyesters, Synthetics</td>
<td>.70</td>
</tr>
<tr>
<td>Cotton, Wool</td>
<td>.70 - .90</td>
</tr>
<tr>
<td>Concrete</td>
<td>.40 - .65</td>
</tr>
<tr>
<td>On Vehicle (Horizontal Movement)</td>
<td>.25 - .40</td>
</tr>
<tr>
<td><strong>Tumbling</strong></td>
<td>.85 - 1.0</td>
</tr>
</tbody>
</table>
**Table 6: Guidelines for Estimating Vehicle Speed Due to Impact**

<table>
<thead>
<tr>
<th>Speed Range</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 Mi/hr or greater</td>
<td>Vehicle surface cleaning or scuffing</td>
</tr>
<tr>
<td>14-15 Mi/hr or greater</td>
<td>Fracture of weight bearing leg of pedestrian</td>
</tr>
<tr>
<td>17 Mi/hr or greater</td>
<td>Deformation of hood</td>
</tr>
<tr>
<td>45 Mi/hr or greater</td>
<td>Gross vehicle deformation</td>
</tr>
<tr>
<td>50-60 Mi/hr</td>
<td>Pedestrian dismemberment</td>
</tr>
</tbody>
</table>

**Table 7: Guidelines for Estimating Vehicle Speed Due to First Head Strike**

*NOTE: Impacts to the pedestrian’s side only. The vehicle must be a pontoon shape.*

<table>
<thead>
<tr>
<th>Speed Range</th>
<th>Impact Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>25-30 Mi/hr</td>
<td>To hood of vehicle</td>
</tr>
<tr>
<td>30-45 Mi/hr</td>
<td>To lower section of windshield</td>
</tr>
<tr>
<td>45-60 Mi/hr</td>
<td>To upper section of windshield</td>
</tr>
<tr>
<td>60 + Mi/hr</td>
<td>To roof section and over</td>
</tr>
</tbody>
</table>

**Table 8: Pedestrian/Vehicle Interrelationships**

<table>
<thead>
<tr>
<th>Kinematic Trajectory</th>
<th>Braking (B)</th>
<th>Non-Braking (NB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wrap Trajectory</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td>Forward Projection</td>
<td>B, NB</td>
<td></td>
</tr>
<tr>
<td>Fender Vault</td>
<td>B, NB</td>
<td></td>
</tr>
<tr>
<td>Roof Vault</td>
<td>NB</td>
<td></td>
</tr>
<tr>
<td>Somersault</td>
<td>B</td>
<td></td>
</tr>
</tbody>
</table>

*SAE #870605*
### 9: Roller blade Skating – Lateral Space

**Lateral Space Used by In-Line Skaters**

<table>
<thead>
<tr>
<th>Skater</th>
<th>Glide Width</th>
<th>Left of Centerline</th>
<th>Right of Centerline</th>
<th>Stride Width</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Novice</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Less than 1 yr</em></td>
<td>3.60 – 4.69 ft</td>
<td>2.03 – 2.19 ft</td>
<td>1.54 – 2.19 ft</td>
<td>3.57 – 4.42 ft</td>
</tr>
<tr>
<td><strong>Intermediate</strong></td>
<td>2.23 – 2.33 ft</td>
<td>1.44 – 2.19 ft</td>
<td>1.08 – 2.98 ft</td>
<td>2.52 – 5.21 ft</td>
</tr>
<tr>
<td><strong>Experienced</strong></td>
<td>2.00 – 2.36 ft</td>
<td>1.31 – 3.15 ft</td>
<td>1.80 – 2.85 ft</td>
<td>3.11 – 5.87 ft</td>
</tr>
</tbody>
</table>

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### 10: Roller blade Skating – Stopping Drag Factor

**Drag Factor Used by In-Line Skaters**

<table>
<thead>
<tr>
<th></th>
<th><strong>Novice</strong></th>
<th><strong>Intermediate</strong></th>
<th><strong>Experienced</strong></th>
<th><strong>Vehicle</strong></th>
</tr>
</thead>
</table>
| **Drag Factor** | Less than 1 yr | 1 – 3 yrs | Experienced |  | 0.69 ABS  
|           | 0.09 – 0.299 | 0.09 – 0.20 | 0.13 – 0.27 | 60 non ABS |

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### 11: Roller blade Skating – Acceleration

**Acceleration Test Results**

<table>
<thead>
<tr>
<th>Accel Units</th>
<th><strong>Novice</strong></th>
<th><strong>Intermediate</strong></th>
<th><strong>Experienced</strong></th>
<th><strong>Vehicle</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><em>ft/sec²</em></td>
<td>Less than 1 yr</td>
<td>1 – 3 yrs</td>
<td>Experienced</td>
<td>11.68 – 12.53</td>
</tr>
<tr>
<td><em>g</em></td>
<td>0.10 – 0.28</td>
<td>0.16 – 0.40</td>
<td>0.25 – 0.44</td>
<td>0.36 – 0.38</td>
</tr>
</tbody>
</table>

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### 12: Roller blade Skating – Velocity

**Average Velocity**

<table>
<thead>
<tr>
<th>Units</th>
<th><strong>Novice</strong></th>
<th><strong>Intermediate</strong></th>
<th><strong>Experienced</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><em>fps</em></td>
<td>Less than 1 yr</td>
<td>1 – 3 yrs</td>
<td>Experienced</td>
</tr>
<tr>
<td><em>mph</em></td>
<td>4.46 – 5.73</td>
<td>7.43 – 9.77</td>
<td>9.81 – 12.5</td>
</tr>
</tbody>
</table>

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1. Minimum speed of a vehicle which is not braking at or prior to impact with a pedestrian, \( \text{mi/hr} \). Center of mass of the pedestrian is required to be at least 3 feet with a sliding friction coefficient of 0.8df.

\[
S_{\text{min}} = \sqrt{58 + 24d_i} - 7.6
\]

Barzeley & Lacy

\( d_i \) = Distance pedestrian thrown from point of impact to point of rest, ft

2. Minimum & maximum velocities of a vehicle at point of impact with a pedestrian, \( \text{ft/sec} \).

\[
\begin{align*}
V &= \sqrt{2\mu gd_i / (\cos \theta + \mu \sin \theta)^2} \\
V_{\text{min}} &= \sqrt{2\mu gd_i / (1 + \mu^2)} \\
V_{\text{max}} &= \sqrt{2\mu gd_i}
\end{align*}
\]

Searle

\( \mu \) = Deceleration factor of pedestrian during slide, decimal

SAE 831622

(recommended 0.66 wet or dry asphalt; 0.79 wet or dry grass)

\( d_i \) = Distance pedestrian thrown from point of impact to point of rest, ft

\( g \) = Gravitational constant, 32.2 ft/sec\(^2\)

\( \theta \) = Pedestrian launch angle, deg

3. Speed of a braking vehicle involved in a pedestrian impact based on throw distance of pedestrian from point of impact to point of rest, \( \text{mi/hr} \). Pre-impact braking required.

\[
S = 6.6\sqrt{8.4f^4 + f d_i} - 20f^2
\]

\( f \) = Deceleration factor of vehicle, decimal (0.7 - 1.2 range)

\( d_i \) = Distance pedestrian thrown from point of impact to point of rest, ft

\[
S = \pm 2.5\text{mph} \quad \text{of rest, ft}
\]

Limpert
4. Maximum speed obtained by a vehicle involved in a pedestrian impact without braking prior to impact, **mi/hr**. Utilize this equation when the operator states they did not observe the pedestrian.

The equation will give you two solutions. Disregard the negative value.

\[
S = \frac{-11 \pm \sqrt{1.21 - 4(1/(30\mu))(-d)}}{2(1/(30\mu))}
\]

*\(\mu\) = Friction coefficient of roadway, decimal
*\(d\) = Post-impact braking distance without skidding from point of impact to point of rest, ft

5. Minimum velocity for a vehicle involved in a pedestrian impact, **ft/sec**. This formula should be used only when the pedestrian is sliding. The pedestrian cannot be tumbling.

\[
V = \left[-z/4 + \sqrt{(z/4)^2 + 4d_s / (2g\mu)}\right]g\mu
\]

*\(z\) = Vertical center of mass height of pedestrian, ft
*\(d\) = Horizontal distance the body traveled from impact to final rest while sliding, ft
*\(\mu\) = Deceleration factor of pedestrian during slide, decimal (Table #4)
*\(g\) = Gravitational constant, 32.2 ft/sec^2

6. Distance a body will travel from point of impact to point of rest, **ft**.

\[
d = \frac{V^2}{2\mu g} (\cos \theta + \mu \times \sin \theta)
\]

*\(\mu\) = Deceleration factor of pedestrian during slide, decimal (Table #4)
*\(V\) = Velocity of vehicle at impact, ft/sec
*\(\theta\) = Pedestrian launch angle, deg
*\(g\) = Gravitational constant, 32.2 ft/sec^2
7. Distance a body will travel from point of impact to point of rest, \textbf{ft}.

\[ d = \frac{S\sqrt{z}}{2.73} + \frac{S^2}{24} \]

\textbf{Collins} \quad S = \text{Speed of vehicle at impact, mi/hr}
\[ z = \text{Vertical center of mass height of pedestrian, ft} \]

8. Velocity of a vehicle from the fall of the pedestrian and slide distance of the vehicle. The following formulas work in conjunction with each other. Both fall and slide calculations should have an equal resultant. These formulas should only be used with a 0° take-off angle and a full vertical impact; the leading edge of the hood of the vehicle must be higher then the center of mass of the pedestrian.

\[ d_f = 2\mu z - 2z\sqrt{\mu^2 - \mu d / z} \]

For fall velocity of pedestrian, \textbf{ft/sec}:

\[ V_f = d_f \sqrt{-g/(2z)} \]

For slide velocity of vehicle, \textbf{ft/sec}:

\[ V_s = \sqrt{2a\left(d - d_f\right)} \]

\textit{Northwestern University, TI/Fricke}

\[ a = \text{Deceleration rate of the pedestrian during slide, ft/sec}^2 \]
\[ \mu = \text{Deceleration factor of pedestrian during slide, decimal (0.45 - 0.6 range)} \]
\[ z = \text{Vertical fall distance of pedestrian’s center of mass, ft} \]
\[ \text{(negative value (-) for a lower center of mass landing)} \]
\[ d = \text{Total distance pedestrian was thrown from point of impact to point of rest, ft} \]
\[ d_f = \text{Horizontal distance the body traveled while falling, ft} \]
\[ d_s = \text{Horizontal distance the body traveled while sliding, ft} \]
\[ g = \text{Gravitational constant, 32.2 ft/sec}^2 \]
9. Distance a body will travel from point of impact to point of rest, ft.

\[ d = \frac{v^2}{2a} + 0.0091va \]

*Stcherbatcheff*  \( V = \) Velocity of vehicle at point of impact, ft/sec

*SAE 751167*  \( a = \) Deceleration rate of the pedestrian during slide, ft/sec²

(13.12 – 22.96 ft/sec² range)

10. Distance a body will travel from point of impact to point of rest, ft.

\[ d = \frac{V^2m_v^2}{2\mu g(m_v + m_p)^2} + \mu z \]

*Wood*  \( V = \) Velocity of vehicle at point of impact, ft/sec

\( m_v = \) Mass of vehicle, lb·sec²/ft

\( m_p = \) Mass of pedestrian, lb·sec²/ft

\( \mu = \) Deceleration factor of pedestrian during slide, decimal

\( g = \) Gravitational constant, 32.2 ft/sec²

\( z = \) Vertical fall distance of pedestrian’s center of mass, ft

11. Minimum velocity of a vehicle at point of impact with a pedestrian, ft/sec.

\[ V_{veh} = \sqrt{\frac{(d - \mu z)2\mu g(m_v + m_p)^2}{m_v}} \]

*Wood*  \( d = \) Distance pedestrian thrown from point of impact to point of rest, ft

\( \mu = \) Deceleration factor of pedestrian during slide, decimal

(recommended 0.66 value)

\( z = \) Vertical center of mass height of pedestrian, ft  (Adjust: \( z-0.5 \))

\( g = \) Gravitational constant, 32.2 ft/sec²

\( m_v = \) Mass of vehicle, lb·sec²/ft

\( m_p = \) Mass of pedestrian, lb·sec²/ft
12. Friction coefficient of a pedestrian during a slide with a known impact speed, \textit{decimal}.

\[ \mu = 0.772 - \frac{S}{1178} \]

\[ S = \text{Speed of vehicle at point of impact, mph} \]

\textit{Wood}

13. Speed of a vehicle at point of impact knowing the pedestrian’s center of mass height, \textit{mi/hr}.

\[ S = \frac{-B + \sqrt{B^2 - 4A(-d)}}{2A} \]

\[ A = \frac{1}{2(\mu g)} \]

\[ B = \frac{\sqrt{z}}{2.73} \]

\textit{Collins}

\[ \mu = \text{Friction coefficient of pedestrian during slide, decimal} \]

\[ z = \text{Center of mass height of pedestrian, ft} \]

\[ d = \text{Distance pedestrian thrown from point of impact to point of rest, ft} \]

14. Velocity of a vehicle at point of impact knowing the distance the pedestrian was thrown from point of impact to point of rest, \textit{ft/sec}.

\[ V = \frac{-B \pm \sqrt{B^2 - 4A(-d_i)}}{2A} \]

\[ A = \frac{1}{2(\mu g)} \]

\[ B = 0.0091\mu g \]

\textit{Stcherbatcheff}

\textit{SAE 751167}

\[ \mu = \text{Pedestrian’s drag factor, decimal} \]

\{0.4 – 0.71 value recommended\}

\[ d_i = \text{Distance pedestrian thrown from point of impact to point of rest, ft} \]

\[ g = \text{Gravitational constant, 32.2 ft/sec}^2 \]
15. Velocity of a vehicle at point of impact knowing the distance the pedestrian slid and height pedestrian fell, **ft/sec**. Forward Projection (full vertical impact: bus, van,...)

\[
V = \frac{-B \pm \sqrt{B^2 - 4A(-d)}}{2A} \quad A = \frac{1}{2g f_p^2} \quad B = \sqrt{2h/g}
\]

\[ f_p = f_v \times 1.14 \rightarrow f_v \times 1.7 (\text{range}) \]

**Eubanks & Dr. Bank**

- \(f_v\) = Deceleration factor of the pedestrian during slide, decimal
- \(h\) = Vertical fall distance of pedestrian's center of mass, ft
- \(g\) = Gravitational constant, 32.2 ft/sec^2
- \(d\) = Distance pedestrian thrown from point of impact to point of rest, ft

16. Speed of a vehicle at point of impact during heavy deceleration, **mph**. The pedestrian must have fallen off the vehicle during the skidding phase. Wrap type trajectory.

\[
S = 5.466 f_p \sqrt{(4t_r + C)^2 + E / f_p} - 4t_r + C
\]

\[
C = \left(1 - f_h / f_p\right) \sqrt{L/(f_v - f_h)} + \sqrt{H_h}
\]

\[
E = (C + \sqrt{H_h}) f_h \sqrt{L/(f_v - f_h)} + d
\]

**Galli**

- \(f_p\) = Friction coefficient of pedestrian sliding on pavement, decimal
- \(f_v\) = Friction coefficient of vehicle sliding on pavement, decimal (0.8 - 1.2 range)
- \(f_h\) = Friction coefficient of pedestrian sliding on hood of vehicle, decimal
- \(t_r\) = Reaction time after impact, sec If pre-impact skid, \(t_r = 0\)
- \(L\) = Distance pedestrian slid on hood commencing from front-end, ft
- \(H_h\) = Height of the leading edge of hood, ft
- \(d\) = Distance pedestrian thrown from point of impact to point of rest, ft
17. Velocity of vehicle at impact knowing the maximum height of pedestrian’s trajectory, \( \text{ft/sec} \).
Do not utilize with forward projection.

\[
V = \frac{8 \sqrt{h_t - h}}{\sin \theta} \quad T = 0.25 \left( \sqrt{h_t - h} + \sqrt{h_t} \right) \quad \theta = \tan^{-1} \left[ \frac{\sqrt{h_t - h}}{\sqrt{16(\mu T)^2 + \mu d_i - 4\mu T}} \right]
\]

Aronberg & Snider  
SAE 900367

\( h_t \) = Maximum height of pedestrian’s trajectory, ft  
\( h \) = Pedestrian’s center of mass height, ft  
\( \mu \) = Pedestrian’s drag factor, decimal (0.5 value recommended)  
\( d_i \) = Throw distance from point of impact to rest, ft  
\( \theta \) = Launch angle of pedestrian, deg  
\( T \) = Pedestrian flight time, sec

18. Velocity of vehicle at impact knowing the distance pedestrian was thrown and the center of mass height, \( \text{ft/sec} \). Do not use if the leading edge is above the pedestrian’s center of mass.

\[
V = 100 \left[ 0.3777 \frac{d_i}{100} + 0.1962(h - l_h)^2 - 0.3051(h - l_h) + 0.3531 \right]
\]

Casteel  
\( d_i \) = Throw distance from point of impact to rest, ft  
\( h \) = Pedestrian’s center of mass height, ft  
\( l_h \) = Height of vehicle’s leading edge, ft

19. Velocity of vehicle at impact knowing the distance pedestrian was thrown and the center of mass height, \( \text{ft/sec} \). The vehicle must have pre-impact braking with the pedestrian coming to rest forward of the vehicle.

\[
V = g \left[ -\sqrt{2z/g} + \sqrt{(2z/g)^2 + (2/g)d_i} \right]
\]

Pultar  
\( d_i \) = Throw distance from point of impact to rest, ft  
\( z \) = Pedestrian’s center of mass height, ft  
\( g \) = Gravitational constant, 32.2 ft/sec\(^2\)
20. Minimum initial velocity of the vehicle at impact knowing the total distance pedestrian was
thrown
from point of impact to rest, **mi/hr**.

\[
\text{Wrap Trajectory, Low Value: } S = 3.30\sqrt{d}
\]

\[
\text{High Value: } S = 4.7\sqrt{d}
\]

\[
\text{Forward Trajectory, Low Value: } S = 3.01\sqrt{d}
\]

\[
\text{High Value: } S = 4.72\sqrt{d}
\]

**Wood**

\[d = \text{Distance pedestrian thrown from point of impact to point of rest, ft Travel on the Vehicle}\]

21. Initial velocity of pedestrian across the path of the vehicle, **ft/sec**.

\[V_a = V_p \sin \theta\]

\[V_p = \text{Pedestrian walking velocity, ft/sec}\]

\[\theta = \text{Approach angle of pedestrian relative to vehicle approach, deg}\]

**Eubanks**

**Note:** Approach angle of pedestrian relative to vehicle approach:
- Walking parallel away or towards vehicle = 0°
- Walking perpendicular (**left to right**) = 90°
- All other approaches fall between 0-90°

22. Time for pedestrian to cross over the hood, **sec**.

\[t_h = \frac{d_h}{V_a}\]

\[d_h = \text{Point initial contact on the vehicle to the point where pedestrian exits, ft}\]

**Eubanks**

\[V_a = \text{Initial velocity of pedestrian across the path of the vehicle, ft/sec (Eq #21)}\]
23. Velocity of a vehicle at point of impact knowing the pre-impact velocity of the pedestrian and angle of approach, \textbf{ft/sec}. Do not utilized if pedestrian is projected forward.

\[ V = -B + \sqrt{\frac{B^2 - 4A(-d)}{2A}} + \frac{l_h}{z}, \quad A = \frac{1}{64.4f_p}, \quad B = \frac{d_h}{V_p \sin \phi} + \sqrt{\frac{h_h}{16.1}} \]

\textit{Eubanks}

\[ f_p = f_v \times 1.14 \rightarrow f_v \times 1.7 \text{(range)} \]

\(f_v\) = Drag factor of vehicle, decimal

\(d\) = Distance pedestrian thrown from point of impact to point of rest, ft

\(d_h\) = Lateral distance from initial contact on the hood of the vehicle by the pedestrian to the point where the pedestrian exits, ft

\(h_h\) = Highest major contact point on the vehicle from the pedestrian measured from the ground, ft

\(l_h\) = Height of vehicle’s leading edge, ft

\(V_p\) = Pre-impact velocity of pedestrian, ft/sec

\(\phi\) = Pedestrian pre-impact direction of travel to the path of the vehicle, deg

(90 deg = perpendicular angle to the vehicle)

\(z\) = Vertical center of mass height of pedestrian, ft

24. Speed of vehicle knowing the total distance pedestrian was thrown, \textbf{mph}.

\textit{Wrap Trajectory:} \[ S = \frac{d}{\sqrt{0.0198}} \]

\textit{Forward Trajectory:} \[ S = \frac{d}{\sqrt{0.0256}} \]

\textit{15 Years of Age or Younger:} \[ S = \frac{d}{\sqrt{0.0268}} \]

\textit{Over the Age of 15:} \[ S = \frac{d}{\sqrt{0.0213}} \]

\textit{Appel} \[ d = \text{Distance pedestrian thrown from point of impact to point of rest, ft} \]
Power

1. \( P = TS \)
   Engine power, \textbf{ft-lb/sec.}

   Gillespie
   \( T = \text{Torque, ft-lb} \)
   \( S = \text{Speed, rad/sec} \)

2. \( H = T\omega_e / 5252 \)
   Horsepower, \textbf{hp.}

   Gillespie
   \( T = \text{Torque, ft-lb} \)
   \( \omega_e = \text{Rotational speed of the engine, rev/min} \)

Radius

A line segment that joins the center of a circle with any point on its circumference.

\[ \pi = \frac{c}{d} \]
\( c = \text{Circumference, ft} \)
\( d = \text{Diameter, ft} \)
2. Radius of a curve knowing the chord and middle ordinate, ft.

\[ r = \frac{C^2}{8Mo} + \frac{Mo}{2} \]

\( C = \) Chord, ft
\( Mo = \) Middle ordinate, ft

3. Radius traveled knowing the track width of the vehicle, chord and middle ordinate of an evident yaw mark, ft.

\[ r = \frac{C^2}{8Mo} + \frac{Mo}{2} - 0.5tw \]

\( C = \) Chord, ft
\( Mo = \) Middle ordinate, ft
\( tw = \) Track width, ft

4. Radius traveled knowing the speed and lateral acceleration factor, ft.

\[ r = \frac{S^2}{15(f_y + e)} \]

\( S = \) Speed, mi/hr
\( f_y = \) Lateral acceleration factor, decimal
\( e = \) Superelevation, maximum 11.9%, decimal
\((-\) value for negative banking)

5. Radius knowing the linear distance along the x-axis and the lateral distance along the y-axis, ft.

\[ r = 0.5\left(\frac{d_x^2}{d_y} + d_y\right) \]

\( d_x = \) Linear distance along x-axis, ft (Eq #9)
\( d_y = \) Lateral distance along y-axis, ft (Eq #11)

6. Radius knowing the linear distance along the x-axis and the lateral distance along the y-axis, ft.

\[ r = \frac{(2d_x)^2}{8d_y} + d_y / 2 \]

\( d_x = \) Linear distance along x-axis, ft (Eq #9)
\( d_y = \) Lateral distance along y-axis, ft (Eq #11)
7. Radius knowing the linear distance along the x-axis and central angle, \( \text{ft} \).

\[
r = d_x / \sin \theta
\]
\( d_x \) = Linear distance along x-axis, \( \text{ft} \) (Eq #9)
\( \theta \) = Central angle, \( \text{deg} \) (Eq #16, 19)

8. Radius knowing the length of the arc and central angle, \( \text{ft} \).

\[
r = d_i / \theta
\]
\( d_i \) = Arc length, \( \text{ft} \)
\( \theta \) = Central angle, \( \text{rad} \)

9. Determining the curvature of the roadway, \( \text{ft} \).

\[
r = \frac{57.3d_i}{\theta}
\]
\( d_i \) = Arc length, \( \text{ft} \)
\( \theta \) = Tangent angle of curve, \( \text{deg} \)

Limpert

10. Linear distance along the x-axis knowing the radius and lateral distance along the y-axis, \( \text{ft} \).

\[
d_x = \sqrt{r^2d_y - d_y^2}
\]
\( d_x \) = Linear distance along x-axis, \( \text{ft} \) (Eq #13)
\( r \) = Radius, \( \text{ft} \)
\( d_y \) = Lateral distance along y-axis, \( \text{ft} \) (Eq #13)

11. Linear distance along the x-axis knowing the radius and central angle, \( \text{ft} \).

\[
d_x = r \sin \theta
\]
\( \theta \) = Central angle, \( \text{deg} \) (Eq #22 thru 24)
\( r \) = Radius, \( \text{ft} \)

12. Linear distance along the x-axis knowing the lateral distance along the y-axis and central angle, \( \text{ft} \).

\[
d_x = d_y / \tan(\theta/2)
\]
\( \theta \) = Central angle, \( \text{deg} \) (Eq #22 thru 24)
\( d_y \) = Lateral distance along y-axis, \( \text{ft} \) (Eq #13)
13 Lateral distance along the y-axis knowing the radius and the central angle, \textbf{ft}.

\[ d_y = r - r \cos \theta \]

\( \theta = \text{Central angle, deg} \) (Eq #14, 15, 18 thru 20)
\( r = \text{Radius, ft} \)

14. Lateral distance along the y-axis knowing the linear distance along the x-axis and central angle, \textbf{ft}.

\[ d_y = d_x \tan \left( \frac{\theta}{2} \right) \]

\( d_x = \text{Linear distance along x-axis, ft} \) (Eq #8)
\( \theta = \text{Central angle, deg} \) (Eq #15, 18 thru 20)

15 Length of an arc knowing the radius and central angle, \textbf{ft}.

\[ d_i = r \pi \left( \frac{\theta}{360} \right) \]

\( r = \text{Radius, ft} \)
\( \pi = \text{Pi, 3.141592654} \)
\( \theta = \text{Central angle, deg} \) (Eq #15 thru 17, 19, 20)

16. Length of an arc knowing the radius and chord, \textbf{ft}.

\[ d_i = r 0.017453 \left( 2 \sin^{-1} \left( 0.5 \frac{C}{R} \right) \right) \]

\( r = \text{Radius, ft} \)
\( C = \text{Chord, ft} \)

17. Length of an arc knowing the radius and central angle, \textbf{ft}.

\[ d_i = r \theta \]

\( r = \text{Radius, ft} \)
\( \theta = \text{Central angle, rad} \)

18. Central angle knowing the radius and length of the arc, \textbf{rad}.

\[ \theta = \frac{d_i}{r} \]

\( r = \text{Radius, ft} \)
\( d_i = \text{Arc length, ft} \)
19. Central angle knowing the radius and linear distance along the x-axis, \(\text{deg.}\)

\[
\theta = \sin^{-1}\left(\frac{d_x}{r}\right) \quad r = \text{Radius, ft} \\
\hspace{1cm} d_x = \text{Linear distance along x-axis, ft (Eq #7)}
\]

20. Central angle knowing the linear distance along the x-axis and lateral distance along the y-axis, \(\text{deg.}\)

\[
\theta = 2\tan^{-1}\left(\frac{d_x}{d_y}\right) \quad d_x = \text{Linear distance along x-axis, ft (Eq #7)} \\
\hspace{1cm} d_y = \text{Lateral distance along y-axis, ft}
\]

21. Central angle knowing the radius and lateral distance along the y-axis, \(\text{deg.}\)

\[
\theta = \cos^{-1}\left(1 - \frac{d_y}{r}\right) \quad r = \text{Radius, ft} \\
\hspace{1cm} d_y = \text{Lateral distance along y-axis, ft}
\]

22. Central angle knowing the radius and length of the arc, \(\text{deg.}\)

\[
\theta = \left(\frac{d_i}{r2\pi}\right)360 \quad r = \text{Radius, ft} \\
\hspace{1cm} \pi = \text{Pi, } 3.141592654 \\
\hspace{1cm} d_i = \text{Arc length, ft}
\]

23. Central angle knowing the chord and middle ordinate of the radius, \(\text{deg.}\)

\[
\theta = 8\tan^{-1}\left(\frac{Mo}{C}\right) \quad C = \text{Chord, ft (Eq #21, 23)} \\
\hspace{1cm} Mo = \text{Middle ordinate, ft (Eq #24)}
\]

24. Central angle knowing the radius and middle ordinate, \(\text{deg.}\)

\[
\theta = 2\cos^{-1}\left(1 - \frac{Mo}{r}\right) \quad r = \text{Radius, ft} \\
\hspace{1cm} Mo = \text{Middle ordinate, ft (Eq #24)}
\]

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25. Determine a chord knowing the radius and middle ordinate, ft.

\[ C = \sqrt{(r - Mo/2)^2} \times Mo \]

\[ r = \text{Radius, ft} \]
\[ Mo = \text{Middle ordinate, ft} \]

26. Determine a chord knowing the radius and middle ordinate, ft.

\[ C = 2\sqrt{Mo(2r - Mo)} \]

\[ r = \text{Radius, ft} \]
\[ Mo = \text{Middle ordinate, ft} \]

27. Determine a chord knowing the middle ordinate and central angle, ft.

\[ C = \frac{Mo}{\tan(\theta/8)} \]

\[ Mo = \text{Middle ordinate, ft} \]
\[ \theta = \text{Central angle, deg (Eq #15 thru 18, 20)} \]

28. Determine a chord knowing the linear distance along the x-axis and lateral distance along the y-axis, ft.

\[ C = \sqrt{d_x^2 + d_y^2} \]

\[ d_x = \text{Linear distance along x-axis, ft (Eq #7 thru 9)} \]
\[ d_y = \text{Lateral distance along y-axis, ft (Eq #10, 11)} \]

29. Determine a middle ordinate knowing the radius and chord of the radius, ft.

\[ Mo = r - \sqrt{r^2 - C^2/4} \]

\[ C = \text{Chord, ft (Eq #23)} \]
\[ r = \text{Radius, ft} \]

30. Determine a middle ordinate knowing the chord of a radius and central angle, ft.

\[ Mo = CTan(\theta/8) \]

\[ C = \text{Chord, ft (Eq #23)} \]
\[ \theta = \text{Central angle, deg (Eq #15 thru 18)} \]
31. Area of a circle knowing the radius, $\text{ft}^2$.

\[ A = r^2 \pi \]

$r = \text{Radius, ft (Eq #30)}$

$\pi = \text{Pi, } 3.141592654$

32. Area of a circle knowing the diameter, $\text{ft}^2$.

\[ A = \frac{d^2 \pi}{4} \]

$d = \text{Diameter, ft (Eq #28)}$

$\pi = \text{Pi, } 3.141592654$

33. Diameter of a circle knowing the circumference, ft.

\[ d = \frac{c}{\pi} \]

$c = \text{Circumference, ft (Eq #29)}$

$\pi = \text{Pi, } 3.141592654$

34. Circumference of a circle knowing the radius, ft.

\[ c = r2 \pi \]

$r = \text{Radius, ft}$

$\pi = \text{Pi, } 3.141592654$

35. Circumference of a circle knowing the diameter, ft.

\[ c = \pi d \]

$d = \text{Diameter, ft}$

$\pi = \text{Pi, } 3.141592654$

36. Radius of a circle knowing the circumference, ft.

\[ r = \frac{c}{(2\pi)} \]

$c = \text{Circumference, ft}$

$\pi = \text{Pi, } 3.141592654$
37. Yaw mark authentication. If the solution is between 0.10 and 0.20 for a deceleration factor, then
the mark is the result of an accurate yaw with no braking applied, decimal.

\[
Y_m = \frac{0.033(S_1^2 - S_2^2)}{d}
\]

\( S_1 = \text{First section of yaw utilized to determine the speed, mi/hr} \)
\( S_2 = \text{Second section of yaw measured from the end of the first chord length, mi/hr} \)
\( d = \text{Distance between the two middle ordinates, ft} \)

38. Radius circumscribed by an oblique or right sided triangle, ft.

\[
R_c = \frac{0.5a}{\sin A} \quad R_c = \frac{0.5b}{\sin B} \quad R_c = \frac{0.5c}{\sin C}
\]

\( a = \text{Hypotenuse (Base), ft} \)
\( b = \text{Side, ft} \)
\( c = \text{Side, ft} \)
\( A = \text{Angle, deg} \)
\( B = \text{Angle, deg} \)
\( C = \text{Angle, deg} \)

39. Radius inscribed by an oblique right sided triangle, ft.

\[
R_i = \sqrt{\frac{(p/2-a)(p/2-b)(p/2-c)}{(p/2)}}
\]

\( a = \text{Hypotenuse (Base), ft} \)
\( b = \text{Side, ft} \)
\( c = \text{Side, ft} \)
\( p = \text{Perimeter, ft (Eq #4)} \)
Railroad Crossing Impacts

**Sight Distance**

G. Sight distance along the railroad tracks to the nearest edge of the considered lane, required for a vehicle to accelerate and clear the tracks, **ft**.

\[
d_T = V_T / V_v \left[ V_v \left( T_p + T_r \right) + \left( V_v^2 / (2gf) \right) + 2d + L_v + w \right]
\]

AASHTO

- \( V_T \) = Velocity of train, **ft/sec**
- \( V_v \) = Velocity of vehicle, **ft/sec**
- \( T_p \) = Perception time, **sec**
- \( T_r \) = Reaction time, **sec**
- \( g \) = Gravitational constant, 32.2 **ft/sec**^2
- \( f \) = Acceleration factor, decimal
- \( d \) = Distance from the stop line or the front of the vehicle to the nearest rail, **ft**
- \( L_v \) = Total length of vehicle, **ft**
- \( w \) = Width between outer rails, **ft**

G. Sight distance along the highway from the nearest rail to the driver of a vehicle, which allows the vehicle to decelerate from a determined velocity in order to stop before the tracks safely with the observation of a train at a determined distance from the crossing, **ft**.

\[
d_H = V_v T_{PR} + V_v^2 / (2gf) + d + d_v
\]

- \( V_v \) = Velocity of vehicle, **ft/sec**
- \( T_{PR} \) = Perception/reaction time, **sec**
  - Assumed: 2.5 seconds

AASHTO

- \( g \) = Gravitational constant, 32.2 **ft/sec**^2
- \( f \) = Deceleration factor, decimal
- \( d \) = Distance from the stop line or the front of the vehicle to the nearest rail, **ft**
- \( d_v \) = Distance from the driver to the front of the vehicle, **ft**
G. Distance required for a vehicle to accelerate from a stop to clear the tracks containing an approaching train, ft.

\[
D = V_T \left( \frac{V_g}{a_1} \right) + \left( \frac{L_V + 2d + w - d_a}{V_g} \right) + T_P + T_R \right) 
\]

\[ V_T = \text{Velocity of train, ft/sec} \]
\[ V_g = \text{Maximum velocity of vehicle in first gear, ft/sec} \]
\[ a_1 = \text{Acceleration factor of vehicle in first gear, ft/sec}^2 \]
\[ L_V = \text{Total length of vehicle, ft} \]
\[ w = \text{Width between outer rails, ft} \]
\[ d = \text{Distance from the stop line or the front of the vehicle to the nearest rail, ft} \]
\[ T_P = \text{Perception time, sec} \]
\[ T_R = \text{Reaction time, sec} \]
\[ d_a = \text{Distance vehicle travels while accelerating to maximum velocity in first gear, ft} \]

**Note:** All values given are on a per axle basis. For tandem suspensions, the value presented is for the average of the two axles.

**Rollover**

**Lateral Stability**

1. Determine a vehicle's lateral stability. The friction coefficient of the roadway must be greater than the value of the solution for the vehicle to rollover, decimal.

\[
f_y = \frac{tw}{2z} \]

\[ tw = \text{Track width, in} \]
\[ z = \text{Vertical center of mass height, in} \]

(Eq #7, 8 Center of Mass section)
**Rollover Probability**

2. Determine a vehicle's rollover probability vs stability factor, **percent**.

\[ R_p = \frac{100}{(1 + f_y^{6.9})} \]

\( f_y = \) Lateral stability, decimal (Eq #1)

_AIQ Issue 20_

**Level Surface**

3. Speed required for a vehicle to rollover from a level surface, **mi/hr**.

\[ S = 0.482 \sqrt{rgtw/ z} \]

\( r = \) Radius traveled by center of mass, ft
\( tw = \) Track width, in
\( z = \) Vertical center of mass height, in
(Eq #7, 8 Center of Mass section)

\( g = \) Gravitational constant, 32.2 ft/sec\(^2\)

4. Velocity required for a vehicle to rollover from a level surface, **ft/sec**.

\[ V = \sqrt{rgtw/2z} \]

\( r = \) Radius traveled by center of mass, ft
\( tw = \) Track width, in
\( z = \) Vertical center of mass height, in
(Eq #7, 8 Center of Mass section)

\( g = \) Gravitational constant, 32.2 ft/sec\(^2\)

5. Velocity required for a vehicle to rollover from a level surface, **ft/sec**.

\[ V = \sqrt{rgf_y} \]

\( f_y = \) Lateral stability, decimal (Eq #1)
\( r = \) Radius traveled by center of mass, ft
\( g = \) Gravitational constant, 32.2 ft/sec\(^2\)

_Woodroofe_
**Banked Surface**

6. Speed required for a vehicle to rollover from a banked surface, **mi/hr**.

\[ S = 3.86 \sqrt{\frac{r(0.5tw + ez)}{z - 0.5twe}} \]

- \( r \) = Radius traveled by center of mass, ft
- \( tw \) = Track width, in
- \( z \) = Vertical center of mass height, in
  (Eq #7, 8 Center of Mass section)
- \( e \) = Elevation, maximum 11.9\%, decimal
  (negative value (-) for decline)

7. Velocity required for a vehicle to rollover from a banked surface, **ft/sec**.

\[ V = \sqrt{\frac{rg(0.5tw + ez)}{z - 0.5twe}} \]

- \( r \) = Radius traveled by center of mass, ft
- \( tw \) = Track width, in
- \( z \) = Vertical center of mass height, in
  (Eq #7, 8 Center of Mass section)
- \( g \) = Gravitational constant, 32.2 ft/sec\(^2\)
- \( e \) = Elevation, maximum 11.9\%, decimal
  (negative value (-) for decline)

**Left side Rollover**

8. Velocity required for a vehicle to rollover to the left knowing center of mass lateral distance from left, **ft/sec**.

\[ V = \sqrt{\frac{rg(1 + z / y_i e)}{z / y_i - e}} \]

- \( r \) = Radius traveled by center of mass, ft
- \( y_i \) = Lateral center of mass distance from left, in
  (Eq #5 Center of Mass section)
- \( z \) = Vertical center of mass height, in
  (Eq #7, 8 Center of Mass section)
- \( g \) = Gravitational constant, 32.2 ft/sec\(^2\)
- \( e \) = Grade or elevation, decimal
  (negative value (-) for decline)
**Right side Rollover**

9. Velocity required for a vehicle to rollover to the right knowing center of mass lateral distance from right, **ft/sec**.

\[ V = \sqrt{\frac{rg(1 + \frac{z}{y_r}e)}{z/y_r - e}} \]

- \( r \) = Radius traveled by center of mass, **ft**
- \( y_r \) = Lateral center of mass distance from right, **in**
  (Eq #6 Center of Mass section)
- \( z \) = Vertical center of mass height, **in**
  (Eq #7, 8 Center of Mass section)
- \( g \) = Gravitational constant, 32.2 ft/sec^2
- \( e \) = Grade or elevation, decimal
  (negative value (-) for decline)

**Rollover**

10. Velocity required for a vehicle to rollover to the left or right knowing center of mass lateral distance, **ft/sec**.

\[ V = \sqrt{R_{\text{center}} \cdot gr} \]

- \( r \) = Radius traveled by center of mass, **ft**
- \( g \) = Gravitational constant, 32.2 ft/sec^2

\[ z = \frac{W_L z_L + W_T z_T}{W} \]

- \( z_L \) = Vertical center of mass height of load from the ground, **in**
- \( z_T \) = Vertical center of mass height of trailer, **in**
- \( W_L \) = Static weight of load, **lb**
- \( W_T \) = Static weight of trailer, **lb**
- \( W \) = Total static weight of semi trailer and load, **lb**

\[ y = z - R_c \]

- \( R_c \) = Roll center from table 2, **in**
\[ x = \frac{yB}{A} \]  
\( B = \) Tire to bed free space, in  
\( A = \frac{1}{2} \) Trailer width, in

\[ tw_a = 0.5tw - x \]  
\( tw_a = \) CM adjustment lateral, in  
\( tw = \) Track width, in

\[ Rt = \frac{tw_a}{z} \]  
\( Rt = \) Rollover Threshold, g’s

\[ Rt_{se} = \frac{Rt + e}{1 - Rt(e)} \]  
\( Rt_{se} = \) Rollover Threshold incorporating Superelevation, g’s  
\( e = \) Superelevation, decimal  
(negative value (-) for decline)

**Tripping, Low Fixed Object**

11. Minimum speed required for a vehicle to rollover after striking a low fixed object such as a curb, **mi/hr**.

\[ S = 5.48\sqrt{z^2 + y^2 - z} \]  
\( y = \) Lateral distance from center of mass to striking side of vehicle, ft (Eq #5, 6 Center of Mass section)  
\( z = \) Vertical center of mass height, ft  
(Eq #7, 8 Center of Mass section)

12. Minimum velocity required for a vehicle to rollover after striking a low fixed object such as a curb, **ft/sec**.

\[ V = 45.5\sqrt{\left[ \frac{I_{roll}}{(Wz)} \right] \sqrt{1 + \left( \frac{tw}{(2z)} \right)^2}} - 1 \]  
\( z = \) Vertical center of mass height, ft  
(Limpert)  
(Eq #7, 8 Center of Mass section)

\( tw = \) Track width, ft  
\( W = \) Total static weight, lb  
\( I_{roll} = \) Roll moment of inertia to the roll axis, ft-lb-sec²  
(Eq #10)
**Inertia, Roll Moment of**

13. Determine the roll moment of inertia to the roll axis, \( \text{ft-lb-sec}^2 \).

\[
I_{\text{roll}} = (I_{\text{roll c.g.}}) + m \left( z^2 + \left( \frac{tw}{2} \right)^2 \right)
\]

\( m = \text{Mass, lb-sec}^2 /\text{ft} \)

\( I_{\text{roll c.g.}} = \text{Roll moment of inertia at the center of mass (NHTSA data), ft-lb-sec}^2 \)

\( z = \text{Vertical center of mass height, ft} \)

(\text{Eq #7, 8 Center of Mass section})

Limpert

\( tw = \text{Track width, ft} \)

**Wind Speed Required, Rollover**

13. Theoretical wind speed required to cause wheel lift or rollover, \( \text{mi/hr} \).

\[
S = \sqrt{\left(0.5Wtw\right) / \left(0.002561Az\right)}
\]

\( W = \text{Gross weight of vehicle, lb} \)

\( tw = \text{Track width, ft} \)

\( A = \text{area of windward side, ft}^2 \)

\( z = \text{Vertical center of mass height, ft} \)

\( (\text{Eq #7, 8}) \)

Ravensdale

**RPM Speed**

**Interior Gear Mechanism**

1. Interior gear-mechanism ratios, \( 00:1 \).

\[
i = \frac{D_N}{D}
\]

\( D_N = \text{Driven gear, \# teeth} \)

\( D = \text{Drive gear, \# teeth} \)

**Final Drive-Gear Ratio**

2. Final drive-gear ratio, \( 00:1 \).

\[
i = \frac{\omega_e r}{168S}
\]

\( \omega_e = \text{Rotational speed of the engine, rev/min} \)

\( S = \text{Speed, mi/hr} \)

\( r = \text{Tire radius, in} \)
3. Final drive gear ratio utilizing interior gear mechanisms, **00:1**.

\[ i = \left( \frac{R_{ws}}{G_{bs}} \right) \left( \frac{C_s}{E_s} \right) \]

- \( R_{ws} \): Rear wheel sprocket, # teeth
- \( G_{bs} \): Gear box sprocket, # teeth
- \( C_s \): Clutch sprocket, # teeth
- \( E_s \): Engine sprocket, # teeth

4. Overall final gear ratio, **00:1**.

\[ i = i_T \cdot i_{TA} \cdot i_A \]

- \( i_T \): Transmission gear ratio, 00:1
- \( i_{TA} \): Auxiliary transmission ratio, 00:1
- \( i_A \): Axle ratio, 00:1 (Differential gear ratio)

**Speed**

5. Speed from transmission measurements, **mi/hr**.

\[ S = \frac{0.00595\omega_r \cdot r}{i_T \cdot i_{TA} \cdot i_A} \]

- \( \omega_r \): Rotational speed of the engine, rev/min
- \( r \): Tire radius, in
- \( i_T \): Transmission gear ratio, 00:1
- \( i_{TA} \): Auxiliary transmission ratio, 00:1
- \( i_A \): Axle ratio, 00:1 (Differential gear ratio)

6. Speed from transmission measurements, **mi/hr**.

\[ S = \frac{\omega_r \cdot r}{i \cdot 168} \]

- \( \omega_r \): Rotational speed of the engine, rev/min
- \( r \): Tire radius, in
- \( i \): Final drive-gear ratio, 00:1 (Eq #3, 4)
**Velocity**

7. \[ V = 0.00436r \left( \frac{\omega_c}{i_T \times i_{TA} \times i_A} \right) \]

Velocity from transmission measurements, ft/sec.

- \( \omega_c \) = Rotational speed of the engine, rev/min
- \( r \) = Tire diameter, in
- \( i_T \) = Transmission gear ratio, 00:1
- \( i_{TA} \) = Auxiliary transmission ratio, 00:1
- \( i_A \) = Axle ratio, 00:1

(Differential gear ratio)

8. Velocity from transmission measurements, ft/sec.

\[ V = \left[ \left( \frac{\omega_c}{(i_T \times i_{TA} \times i_A)} \right) (r \times 2\pi / 12) \right] / 60 \]

\( \omega_c \) = Rotational speed of the engine, rev/min

- \( r \) = Tire radius, in
- \( i_T \) = Transmission gear ratio, 00:1
- \( i_{TA} \) = Auxiliary transmission ratio, 00:1
- \( i_A \) = Axle ratio, 00:1

(Differential gear ratio)

\( \pi \) = Pi, 3.141592654

**RPM**

9. Determine the rotational speed of a vehicle's engine from the gear ratio, rev/min.

\[ \omega_c = \frac{S \times i \times 168}{r} \]

- \( S \) = Speed, mi/hr
- \( r \) = Tire radius, in
- \( i \) = Final drive-gear ratio, 00:1 (Eq #3, 4)
Drive Wheel Radius

10. Radius of the drive wheel from transmission measurements, \textbf{in}.

\[
\frac{S \times i \times 168}{\omega_e} = r
\]

\(\omega_e = \text{Rotational speed of the engine, rev/min}\)

\(S = \text{Speed, mi/hr}\)

\(i = \text{Final drive-gear ratio, 00:1 (Eq #3, 4)}\)

11. Velocity from transmission measurements incorporating air resistance, \textbf{ft/sec}.

\[
V = \sqrt{\frac{i_r i_A n (T_e / R) - f_{roll} W}{C_D A (\rho / 2)}}
\]

\(i_r = \text{Transmission gear ratio, 00:1}\)

\(i_A = \text{Axle ratio, 00:1}\)

\(n = \text{Mechanical efficiency of drive train, decimal}\)

\(T_e = \text{Torque at maximum rpm, ft/lb}\)

\(R = \text{Radius of drive wheel, ft}\)

\(f_{roll} = \text{Rolling resistance coefficient, decimal}\)

\(W = \text{Weight of vehicle, lb}\)

\(C_D = \text{Aerodynamic drag coefficient, decimal}\)

\(A = \text{Vehicle frontal area, ft}^2\)

\(\rho = \text{Mass density of air, lb.sec}^2 /\text{ft}^4\)

Speed

The rate or a measure of the rate of motion, especially: Distance traveled divided by the time of travel. The limit of this quotient as the time of travel becomes vanishingly small; the first derivative of distance with respect to time. The magnitude of a velocity.

1. Equivalent constant speed from a known velocity, \textbf{mi/hr}.

\[
S = \frac{V}{1.466}
\]

\(V = \text{Velocity constant, ft/sec}\)
2. Equivalent constant speed from a known velocity, **mi/hr**.

\[ S = V(0.682) \]

\( V = \text{Velocity constant, ft/sec} \)

3. Equivalent constant speed from a known velocity, **mi/hr**.

\[ S = V \left( \frac{60^2}{5280} \right) \]

\( V = \text{Velocity constant, ft/sec} \)

4. Constant speed over a determined distance and a unit of time, **mi/hr**.

\[ S = \frac{d}{1.466T} \]

\( d = \text{Distance, ft} \)

\( T = \text{Time, sec} \)

5. Initial speed of a deceleration to a stop over a determined distance and a unit of time, **mi/hr**.

\[ S = \frac{2d}{1.466T} \]

\( d = \text{Distance, ft} \)

\( T = \text{Time, sec} \)

6. Initial speed of a deceleration to a stop on a surface grade less than 11.9 %, **mi/hr**.

\[ S = \sqrt{30d(\mu n + m)} \]

\( d = \text{Distance, ft} \)

\( \mu = \text{Level friction coefficient, decimal} \)

\( m = \text{Grade, maximum 11.9\%, decimal} \)

\( n = \text{Braking efficiency, decimal} \)

\( (+) \text{ for incline, } (-) \text{ for decline} \)
7. Initial speed of a deceleration to a stop on a surface grade greater than 6.8°, \text{mi/hr}.

\[ S = \sqrt{30d(\mu \cos \theta + \sin \theta)n} \]

\( d \) = Distance, ft
\( \mu \) = Friction coefficient, decimal
\( \theta \) = Angle of grade, deg (negative value (-) for decline)
\( n \) = Braking efficiency, decimal

8. Initial speed of a deceleration on two different surfaces at the same time with 100% braking, \text{mi/hr}. Ex. left side tires on roadway, right side tires off roadway.

\[ S = \sqrt{15d(\mu_l + \mu_r)} \]

\( d \) = Distance, ft
\( \mu_l \) = Friction coefficient, left side, decimal
\( \mu_r \) = Friction coefficient, right side, decimal

9. Initial speed of a deceleration on two different surfaces at the same time with different braking efficiency for each side, \text{mi/hr}. Ex; left side tires on roadway, right side tires off roadway.

\[ S = \sqrt{30d(\mu_l n_l + \mu_r n_r)} \]

\( d \) = Distance, ft
\( \mu_l \) = Friction coefficient, left side, decimal
\( \mu_r \) = Friction coefficient, right side, decimal
\( n_l \) = Braking efficiency, left side maximum value of 50%, decimal
\( n_r \) = Braking efficiency, right side maximum value of 50%, decimal
10. Initial speed of a deceleration on two different surfaces at the same time incorporating a grade with different braking efficiency for each side, **mi/hr**. Ex; left side tires on roadway, right side tires off roadway.

\[
S = \sqrt{30d((\mu_i n_i + \mu_r n_r) \pm m)}
\]

- \(d\) = Distance, ft
- \(\mu_i\) = Friction coefficient, left side, decimal
- \(\mu_r\) = Friction coefficient, right side, decimal
- \(n_i\) = Braking efficiency, left side maximum value of 50%, decimal
- \(n_r\) = Braking efficiency, right side maximum value of 50%, decimal
- \(m\) = Grade, maximum 11.9%, decimal \((+)\) for incline, \((-)\) for decline

11. Combined speed of a deceleration over several surfaces, **mi/hr**.

\[
S = \sqrt{S_1^2 + S_2^2 + S_3^2 + \ldots + S_n^2}
\]

- \(S_i \rightarrow S_n\) = Pre-determined speeds of individual surfaces, **mi/hr**

12. Combined speed of a deceleration over several surfaces, **mi/hr**.

\[
S = \sqrt{30(\mu_1 d_1 n_1 + \mu_2 d_2 n_2 + \ldots + \mu_n d_n n_n)}
\]

- \(d_i \rightarrow d_n\) = Distance of individual surfaces, ft
- \(\mu_i \rightarrow \mu_n\) = Friction coefficient of individual surfaces, decimal
- \(n_i \rightarrow n_n\) = Braking efficiency for individual surfaces, decimal

13. Speed gained or lost after accelerating/decelerating over a unit of time, **mi/hr**.

\[
S = 0.682aT
\]

- \(a\) = Accel / Decel rate, \(ft/sec^2\)
- \(T\) = Time, sec
14. Speed gained or lost after accelerating/decelerating over a unit of time, \textit{mi/hr}.

\[ S = 21.96fT \]

\( f = \text{Accel / Decel factor, decimal} \)
\( T = \text{Time, sec} \)

15. Final speed after accelerating/decelerating over a unit of time, \textit{mi/hr}.

\[ S_f = So + 21.96fT \]

\( So = \text{Speed initial, mi/hr} \)
\( T = \text{Time, sec} \)
\( f = \text{Accel / Decel factor, decimal} \)
(negative value (-) for deceleration)

16. Final speed after accelerating/decelerating over a determine distance on a surface grade less than 6.8° (11.9 %), \textit{mi/hr}.

\[ S_f = \sqrt{So^2 + 30dfn} \]

\( So = \text{Speed initial, mi/hr} \)
\( d = \text{Distance, ft} \)
\( f = \text{Accel / Decel factor, decimal} \)
(negative value (-) for deceleration)
\( n = \text{Braking efficiency, decimal (deceleration only)} \)

17. Speed of a vehicle from a critical speed scuff with a known traveled radius at the center of mass, \textit{mi/hr}. Superelevation is less than 6.8° (11.9 %).

\[ S = \sqrt{15r(f_y + e)} \]

\( r = \text{Radius traveled by center of mass, ft} \)
\( f_y = \text{Lateral acceleration factor, decimal} \)
\( e = \text{Superelevation, maximum 11.9%, decimal} \)
((-) value for negative banking)}
18. Speed of a vehicle from a critical speed scuff with an unknown traveled radius at the center of mass, \( \text{mi/hr} \). Superelevation is less than 6.8° (11.9%).

\[
S = \sqrt{15(r - 0.5tw)(f_y + e)}
\]

- \( r \) = Radius of yaw mark, ft
- \( tw \) = Track width, ft
- \( f_y \) = Lateral acceleration factor, decimal
- \( e \) = Superelevation, maximum 11.9%, decimal
  \((\cdot) \text{ value for negative banking}\)

19. Speed of a vehicle from a critical speed scuff with an unknown traveled radius at the center of mass, \( \text{mi/hr} \). Superelevation is either positive or negatively banked greater than 6.8° (11.9%).

\[
S = \sqrt{\frac{15(r - 0.5tw)(f_y + e)}{1 - f_y e}}
\]

- \( r \) = Radius of yaw mark, ft
- \( tw \) = Track width, ft
- \( f_y \) = Lateral acceleration factor, decimal
- \( e \) = Superelevation, minimum 11.9%, decimal
  \((\cdot) \text{ value for negative banking}\)

20. Determine a speed knowing the kinetic energy generated and weight of the object, \( \text{mi/hr} \).

\[
S = \sqrt{\frac{30E}{W}}
\]

- \( E \) = Kinetic energy, ft-lb
- \( W \) = Total static weight, lb

**Hydroplane**

21. Minimum speed required to hydroplane with tires which have normal wear, \( \text{mi/hr} \). The depth of the water on the surface must exceed the tread depth of tires across any parallel point before a vehicle will hydroplane. There must be at least 1/5 of an inch of water depth.  

\[
S = 10.35\sqrt{p}
\]

- \( p \) = Front-tire inflation pressure, psi

*Horne/Joyner*

*SAE 650145*
22. Front-tire inflation pressure required to hydroplane at a given speed, **psi**.

\[
p = \left( \frac{S}{10.35} \right)^2
\]

\[S = \text{Speed, mi/hr}\]

_Horne/Joyner_

_SAЕ 650145_

23. Speed required to hydroplane with tires which are badly worn to nearly bald, **mi/hr**. The depth of the water does not need to exceed the tread depth before a vehicle will hydroplane.

\[
S = 9.03 \sqrt{\frac{p}{w/L}}
\]

\[p = \text{Front-tire inflation pressure, lb/in}^2\]

\[w = \text{Width of tire contact patch, in}\]

\[L = \text{Length of tire contact patch, in}\]

_Horne/Joyner_

_SAЕ 650145_

24. Speed required to hydroplane for a non-articulated vehicle, **mi/hr**. The depth of the water on the surface must exceed the tread depth of tires across any parallel point before a vehicle will hydroplane. There must be at least 1/5 of an inch of water depth.

\[
S = 21.19 p^{0.3} \sqrt{\frac{w}{L}}
\]

\[p = \text{Front-tire inflation pressure, psi}\]

\[w = \text{Width of tire contact patch, in}\]

\[L = \text{Length of tire contact patch, in}\]

_Navin_
25. Speed required to hydroplane for commercial motor vehicles which is lightly laded, \( \text{mi/hr} \). The depth of the water on the surface must exceed the tread depth of tires across any parallel point before a vehicle will hydroplane. There must be at least 1/5 of an inch of water depth.

\[
S = 7.95\sqrt{\frac{p}{(w/L)}}
\]

\( p = \) Front-tire inflation pressure, psi
\( w = \) Width of tire contact patch, in
\( L = \) Length of tire contact patch, in

Dunlap

26. Speed required to hydroplane for commercial motor vehicles which is lightly laded, \( \text{mi/hr} \). The depth of the water on the surface must exceed the tread depth of tires across any parallel point before a vehicle will hydroplane. There must be at least 1/5 of an inch of water depth.

\[
S = 27.40 p^{0.21} \sqrt{(w/L)}^{-1}
\]

\( p = \) Front-tire inflation pressure, psi
\( w = \) Width of tire contact patch, in
\( L = \) Length of tire contact patch, in

Ivey

27. Maximum speed possible in order to stop from a known distance; (hill crest, bend in roadway) when first perception of an obstacle occurs, \( \text{mi/hr} \).

\[
S = 21.96 f \left[ \sqrt{T^2 + 0.0621d/f} - T \right]
\]

\( f = \) Deceleration factor, decimal
\( d = \) Total distance to Impact, ft
\( (\text{including P/R distance}) \)
\( T = \) Perception/Reaction time, sec
28. Speed accounting for weight shift during a deceleration, \textbf{mi/hr}.

\[ S = \sqrt{\frac{30}{W} \left[ f_F d_F (W_F + \Delta W) + f_R d_R (W_R - \Delta W) + dm \right]} \]

- \( d \): Total distance center of mass traveled, ft
- \( d_F \): Front axle skid distance, ft
- \( d_R \): Rear axle skid distance, ft
- \( f_F \): Front deceleration factor, decimal
- \( f_R \): Rear deceleration factor, decimal
- \( W \): Total static weight, lb
- \( W_F \): Static weight of front axle, lb
- \( W_R \): Static weight of rear axle, lb
- \( \Delta W \): Weight shift to front axle, lb (Eq #1, 2, 3 Weight Shift section)
- \( m \): Grade, maximum 11.9\%, decimal (negative value (-) for decline)

29. Yaw mark authentication. If the solution is between 0.10 and 0.20 for a deceleration factor, then the mark is the result of an accurate yaw with no braking applied, \textbf{decimal}.

\[ Y_m = \frac{0.033 (S_1^2 - S_2^2)}{d} \]

- \( S_1 \): First section of yaw utilized to determine the speed, mi/hr
- \( S_2 \): Second section of yaw measured from the end of the first chord length, mi/hr
- \( d \): Distance between the two middle ordinates, ft
Spin Out

To utilize the below equations, the wheels must be free to roll and the linear spinout distance is relatively long. Braking should be less than 100%; however, this does not apply to sliding tires during side slip in the course of the spin.

1. Force at a determined point in a spin, \( \text{lb} \).

\[
F = (\mu \sin \theta + \mu n \cos \theta)W
\]

- \( F \) = Total static weight of vehicle, \( \text{lb} \)
- \( \mu \) = Friction coefficient, decimal
- \( \theta \) = Angle of rotation at a point in the spin, \( \text{deg} \)
- \( n \) = Braking efficiency, decimal

2. Energy dissipated from the beginning point in a phase of a spin to the next measured point, \( \text{ft-lb} \).

\[
E = \left( \frac{F_1 + F_2}{2} \right)(d_2 - d_1)
\]

- \( F_1 \) = Force generated at previous point in the spin, \( \text{lb} \) (Eq #1)
- \( F_2 \) = Force generated at next immediate point in the spin, \( \text{lb} \) (Eq #1)
- \( d_1 \) = Distance to first point \( (F_1) \) in phase being measured from point zero, \( \text{ft} \)
- \( d_2 \) = Distance to next point \( (F_2) \) in phase being measured from point zero, \( \text{ft} \)

3. Initial speed at commencement of spinout, \( \text{mi/hr} \).

\[
S = \sqrt{\frac{30E_T}{W}}
\]

- \( E_T \) = Total energy dissipated from spin, \( \text{ft-lb} \)
- \( W \) = Total static weight of vehicle, \( \text{lb} \)
4. Initial velocity at commencement of spinout, \( \text{ft/sec} \).

\[
V = \frac{\sqrt{2gE_T}}{W}
\]

\( E_T = \) Total energy dissipated from spin, \( \text{ft-lb} \)
\( W = \) Total static weight of vehicle, \( \text{lb} \)
\( g = \) Gravitational constant, \( 32.2 \text{ ft/sec}^2 \)

5. Average deceleration factor from the spin, \( \text{decimal} \).

\[
f = \frac{S^2}{30d}
\]

\( S = \) Initial speed at the commencement of the spinout, \( \text{mi/hr} \) (Eq \#3)
\( d = \) Total linear distance of spin, \( \text{ft} \)

6. Average deceleration factor from the spin, \( \text{decimal} \).

\[
f = \frac{V^2}{2gd}
\]

\( V = \) Initial velocity at the commencement of the spinout, \( \text{ft/sec} \) (Eq \#4)
\( d = \) Total linear distance of spin, \( \text{ft} \)
\( g = \) Gravitational constant, \( 32.2 \text{ ft/sec}^2 \)

**Equivalent Friction Coefficient**

N1. Drag factor of the rolling tires, along their \( x\)-axis at a point in the spin, \( \text{decimal} \).

(Original direction of travel)

\[
f_1 = \cos \theta
\]

\( \theta = \) Angle of rotation measured at a point in the spin from the vehicle's original direction of travel, \( \text{deg} \).

N2. Drag factor of tires in the direction of the side slip at point in the spin, \( \text{decimal} \).

\[
f_2 = \sin \theta
\]

\( \theta = \) Angle of rotation measured at a point in the spin from the vehicle's original direction of travel, \( \text{deg} \).
N3. Average rolling drag factor, decimal.

\[ f_{1A} = f_1 \mu_r \]

\( f_1 \) = Overall average rolling drag factor summed from each point in the spin, decimal (Eq #N1)
\( \mu_r \) = Rolling friction coefficient, decimal (Drag Coefficients)

N4. Average side slip drag factor, decimal.

\[ f_{2A} = f_2 \mu \]

\( f_2 \) = Overall average side slip drag factor summed from each point in the spin, decimal
\( \mu \) = Friction coefficient, decimal

N5. Combined drag factor, decimal.

\[ f_c = f_{1A} + f_{2A} \]

\( f_{1A} \) = Average rolling drag factor, decimal (Eq #N3)
\( f_{2A} \) = Average side slip drag factor, decimal (Eq #N4)

N6. Minimum speed at the commencement of a spin utilizing an equivalent deceleration factor, mi/hr.

\[ S = \sqrt{30 f_c d} \]

\( f_c \) = Combined equivalent drag factor, decimal (Eq #N5)
\( d \) = Total linear distance in spin, ft
Tangent Offset

The trigonometric function of an acute angle in a right triangle that is the ratio of the length of the side opposite the angle to the length of the side adjacent to the angle.

1. Perpendicular offset length from any measured distance along a baseline to plot a radius, ft.

\[ h = r - \sqrt{r^2 - d^2} \]

\( r = \) Radius, ft
\( d = \) Distance along baseline, ft

2. Perpendicular offset length from any measured distance along a baseline to plot a radius, ft.

\[ h = \frac{d^2}{2r} \]

\( r = \) Radius, ft
\( d = \) Distance along baseline, ft

3. Individual measured offset locations along a baseline from a pre-determined total amount, ft.

\[ h = r - \sqrt{r^2 - \left(\frac{I_o r}{N}\right)^2} \]

\( r = \) Radius, ft
\( N = \) Total number of offsets utilized, #
\( I_o = \) Numbered offset along baseline, # (1, 2, 3, 4,......)

4. Determine a radius knowing the tangent offset location and distance along the baseline, ft.

\[ r = \frac{\left(h^2 + d^2\right)}{2h} \]

\( h = \) Offset length from baseline, ft
\( d = \) Distance along baseline, ft

5. Distance along the baseline knowing the offset length and radius, ft.

\[ d = \sqrt{h^2 r - h} \]

\( h = \) Offset length from baseline, ft
\( r = \) Radius, ft
**Time**

*Time* is a non-spatial continuum in which events occur in apparently irreversible succession from the past through the present to the future. An interval separating two points on this continuum; a duration.

1. **Time of travel at a constant velocity over a determined distance, sec.**

\[ T = \frac{d}{V} \]

- \(d\) = Distance, ft
- \(V\) = Velocity constant, ft/sec

2. **Time of travel at a constant speed over a determined distance, sec.**

\[ T = \frac{d}{(1.466S)} \]

- \(d\) = Distance, ft
- \(S\) = Speed constant, mi/hr

3. **Time required to accel/decelerate from or to a stop, sec.**

\[ T = \frac{V}{(\text{unsigned})} \]

- \(V\) = Velocity, ft/sec
- \(\mu\) = Friction coefficient, decimal
- \(g\) = Gravitational constant, 32.2 ft/sec²
- \(n\) = Braking efficiency, decimal (deceleration only)

4. **Time required to accel/decelerate from or to a stop, sec.**

\[ T = \frac{V}{a} \]

- \(V\) = Velocity, ft/sec
- \(a\) = Accel / Decel rate, ft/sec²

5. **Time required to accel/decelerate from or to a stop, sec.**

\[ T = 0.25 \sqrt{\frac{d}{(\mu n \pm m)}} \]

- \(d\) = Distance, ft
- \(\mu\) = Level friction coefficient, decimal
- \(n\) = Braking efficiency, decimal (deceleration only)
- \(m\) = Grade, maximum 11.9%, decimal
  - \((+\) for incline, (-) for decline)
6. Time required to accel/decelerate from or to a stop, sec.

\[ T = \sqrt{\frac{d}{0.5fg}} \]

- \( d \) = Distance, ft
- \( f \) = Accel / Decel factor, decimal
- \( g \) = Gravitational constant, 32.2 ft/sec\(^2\)

7. Time required to accel/decelerate from or to a stop, sec.

\[ T = \sqrt{\frac{d}{0.5a}} \]

- \( d \) = Distance, ft
- \( a \) = Accel / Decel rate, ft/sec\(^2\)

8. Time required to accel/decelerate from or to a stop, sec.

\[ T = \frac{2d}{V} \]

- \( d \) = Distance, ft
- \( V \) = Velocity, ft/sec

9. Time required to accel/decelerate from or to a stop, sec.

\[ T = \frac{1.36d}{S} \]

- \( d \) = Distance, ft
- \( S \) = Speed, mi/hr

10. Time required to accel/decelerate from or to a stop, sec.

\[ T = \sqrt{\frac{2d}{a}} \]

- \( d \) = Distance, ft
- \( a \) = Accel / Decel rate, ft/sec\(^2\)

11. Time required to accel/decelerate from or to a stop, sec.

\[ T = \sqrt{\frac{d}{(\mu gn/2)}} \]

- \( d \) = Distance, ft
- \( \mu \) = Friction coefficient, decimal
- \( g \) = Gravitational constant, 32.2 ft/sec\(^2\)
- \( n \) = Braking efficiency, decimal (deceleration only)
12. Time required to accel/decelerate from or to a stop, \textbf{sec.}

\[ T = \frac{S}{(21.96 f)} \]

\( S = \text{Speed, mi/hr} \)

\( f = \text{Accel / Decel factor, decimal} \)

13. Time required to decelerate from one speed to another, \textbf{sec.}

\[ T = \frac{(S_o - S_f)}{(21.96 f)} \]

\( S_o = \text{Speed initial, mi/hr} \)

\( S_f = \text{Speed final, mi/hr} \)

\( f = \text{Deceleration factor, decimal} \)

14. Time required to accelerate from one speed to another, \textbf{sec.}

\[ T = \frac{(S_f - S_o)}{(21.96 f)} \]

\( S_f = \text{Speed final, mi/hr} \)

\( S_o = \text{Speed initial, mi/hr} \)

\( f = \text{Acceleration factor, decimal} \)

15. Time required to decelerate from one velocity to another, \textbf{sec.}

\[ T = \frac{V_o - V_f}{fg} \]

\( V_o = \text{Velocity initial, ft/sec} \)

\( V_f = \text{Velocity final, ft/sec} \)

\( f = \text{Deceleration factor, decimal} \)

\( g = \text{Gravitational constant, 32.2 ft/sec}^2 \)

16. Time required to decelerate from one velocity to another, \textbf{sec.}

\[ T = \frac{(V_o - V_f)}{a} \]

\( V_o = \text{Velocity initial, ft/sec} \)

\( V_f = \text{Velocity final, ft/sec} \)

\( a = \text{Deceleration rate, ft/sec}^2 \)
17. Time required to accelerate from one velocity to another, \textbf{sec}.

\[ T = \frac{V_f - V_0}{f g} \]

\( V_f = \) Velocity final, ft/sec
\( V_0 = \) Velocity initial, ft/sec
\( f = \) Acceleration factor, decimal
\( g = \) Gravitational constant, 32.2 ft/sec\(^2\)

18. Time required to accelerate from one velocity to another, \textbf{sec}.

\[ T = \frac{(V_f - V_0)}{a} \]

\( V_f = \) Velocity final, ft/sec
\( V_0 = \) Velocity initial, ft/sec
\( a = \) Acceleration rate, ft/sec\(^2\)

19. Time required to decelerate from an initial velocity over a determined distance, \textbf{sec}.

\[ T = \frac{V_0 - \sqrt{V_0^2 - 2d \mu g n}}{\mu g n} \]

\( V_0 = \) Velocity initial, ft/sec
\( d = \) Distance, ft
\( \mu = \) Friction coefficient, decimal
\( g = \) Gravitational constant, 32.2 ft/sec\(^2\)
\( n = \) Braking efficiency, decimal

20. Time required to accelerate from an initial velocity over a determined distance, \textbf{sec}.

\[ T = \frac{-V_0 + \sqrt{V_0^2 + 2d f g}}{f g} \]

\( V_0 = \) Velocity initial, ft/sec
\( d = \) Distance, ft
\( f = \) Acceleration factor, decimal
21. Time required to decelerate from one velocity to another over a determined distance, sec.

\[ T = \frac{2d}{(V_0 + V_f)} \]

- \( Vo \) = Velocity initial, ft/sec
- \( V_f \) = Velocity final, ft/sec
- \( d \) = Distance, ft

22. Time of flight for a vehicle, which has gone airborne, sec.

\[ T = \frac{d}{(V \cos \theta)} \]

- \( d \) = Horizontal distance center of mass traveled from take-off to landing, ft
- \( V \) = Velocity, ft/sec
- \( \theta \) = Angle of take-off, deg

23. Time of flight for a vehicle, which has gone airborne, sec.

\[ T = \frac{-VSin\theta - \sqrt{(VSin\theta)^2 - h^2g}}{-g} \]

- \( V \) = Velocity, ft/sec
- \( h \) = Vertical distance from the plane of take-off to landing, ft (negative value (-) for a lower center of mass landing)
- \( \theta \) = Angle of take-off, deg
- \( g \) = Gravitational constant, 32.2 ft/sec²

24. Time of flight from take-off to maximum vertical height for a vehicle which has gone airborne, sec.

\[ T_m = \frac{VSin \theta}{g} \]

- \( V \) = Velocity, ft/sec
- \( \theta \) = Angle of take-off, deg

\[ g = \text{Gravitational constant, } 32.2 \text{ ft/sec}^2 \]
25. Time from maximum vertical height to landing for a vehicle which has gone airborne, sec.

\[ T_L = \frac{\sqrt{2h_z}}{g} \]

\( h_z = \text{Distance from maximum vertical height to landing, ft (Eq #28 Airborne section)} \)

\( g = \text{Gravitational constant, 32.2 ft/sec}^2 \)

25. Time of rotation after impact, sec.

\[ T = \frac{2\theta}{\omega} \]

\( \theta = \text{Total degree of rotation, rad} \)

\( \omega = \text{Angular velocity, rad/sec (Eq #14 Damage (Crush) section)} \)

27. Change in time during a collision (duration of impact; typically 0.1 - 0.2 seconds), sec.

\[ \Delta T = \frac{P}{F} \]

\( P = \text{Impulse, lb-sec (Eq #23 Damage (Crush) section)} \)

\( F = \text{Collision force at the centroid of damage, lb (Eq #19 Damage (Crush) section)} \)

Perception/Reaction

28. Perception/Reaction time knowing the initial velocity, total time & distance traveled, and deceleration factor, Sec.

\[ T_{PR} = \left( \frac{d - 16.1 ft^2}{V_0} \right) - T \]

\( V_0 = \text{Velocity initial, ft/sec} \)

\( d = \text{Total distance traveled, ft} \)

\( T = \text{Total time, sec} \)

\( f = \text{Deceleration factor, decimal \{negative value (-) for deceleration\}} \)
29. Perception/Reaction time knowing the initial and final velocities, total time & distance traveled, and deceleration factor, \textbf{Sec.}

\[
T_{PR} = \frac{1}{V_o} \left( d - \frac{V_f^2 - V_o^2}{2gf} \right)
\]

- \(V_o\) = Velocity initial, ft/sec
- \(V_f\) = Velocity final, ft/sec
- \(d\) = Total distance traveled, ft
- \(T\) = Total time, sec
- \(f\) = Deceleration factor, decimal
  \{(negative value (-) for deceleration)\}
- \(g\) = Gravitational constant, 32.2 ft/sec$^2$

\section*{Tires}

\textit{Tire Diameter}

1. Determine the tire diameter based on tire measurements, \textbf{ft}.

\textbf{Example:} \textit{255/70R15}

\begin{align*}
255 \times 0.70 &= 178.5 \\
178.5 \times 0.03937 &= 7.027545 \\
7.027545 + (15 \times 0.5) &= 14.527545 \\
14.527545 / 12 &= 1.21062875 \\
\end{align*}

\textit{Tire radius} = 1.21062875 feet

\textit{Tire diameter} = 2.4212575 feet
**Trigonometry**

*The branch of mathematics that deals with the relationships between the sides and angles of triangles and with the properties and applications of the trigonometric functions of angles.*

\[
\text{Sine} = \frac{\text{Opposite}}{\text{Hypotenuse}} \quad \text{Cotangent} = \frac{\text{Opposite}}{\text{Adjacent}}
\]

\[
\text{Cosine} = \frac{\text{Adjacent}}{\text{Hypotenuse}} \quad \text{Tangent} = \frac{\text{Opposite}}{\text{Adjacent}}
\]

**Table 1; Right Sided Triangle**

<table>
<thead>
<tr>
<th>Known Sides &amp; Angles</th>
<th>Formulas for Unknown Sides and Angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sides a and b</td>
<td>$c = \sqrt{a^2 - b^2}$</td>
</tr>
<tr>
<td></td>
<td>$\sin B = b / a$</td>
</tr>
<tr>
<td></td>
<td>$C = 90^\circ - B$</td>
</tr>
<tr>
<td>Sides a and c</td>
<td>$b = \sqrt{a^2 - c^2}$</td>
</tr>
<tr>
<td></td>
<td>$\sin C = c / a$</td>
</tr>
<tr>
<td></td>
<td>$B = 90^\circ - C$</td>
</tr>
<tr>
<td>Sides b and c</td>
<td>$a = \sqrt{b^2 + c^2}$</td>
</tr>
<tr>
<td></td>
<td>$\tan B = b / c$</td>
</tr>
<tr>
<td></td>
<td>$C = 90^\circ - B$</td>
</tr>
<tr>
<td>Side a; angle B</td>
<td>$b = a \cdot \sin B$</td>
</tr>
<tr>
<td></td>
<td>$c = a \cdot \cos B$</td>
</tr>
<tr>
<td></td>
<td>$C = 90^\circ - B$</td>
</tr>
<tr>
<td>Side a; angle C</td>
<td>$b = a \cdot \cos C$</td>
</tr>
<tr>
<td></td>
<td>$c = a \cdot \sin C$</td>
</tr>
<tr>
<td></td>
<td>$B = 90^\circ - C$</td>
</tr>
<tr>
<td>Side b; angle B</td>
<td>$a = b / \sin B$</td>
</tr>
<tr>
<td></td>
<td>$c = b / \tan B$</td>
</tr>
<tr>
<td></td>
<td>$C = 90^\circ - B$</td>
</tr>
<tr>
<td>Side b; angle C</td>
<td>$a = b / \cos C$</td>
</tr>
<tr>
<td></td>
<td>$c = b \cdot \tan C$</td>
</tr>
<tr>
<td></td>
<td>$B = 90^\circ - C$</td>
</tr>
<tr>
<td>Side c; angle B</td>
<td>$a = c / \cos B$</td>
</tr>
<tr>
<td></td>
<td>$b = c \cdot \tan B$</td>
</tr>
<tr>
<td></td>
<td>$C = 90^\circ - B$</td>
</tr>
<tr>
<td>Side c; angle C</td>
<td>$a = c / \sin C$</td>
</tr>
<tr>
<td></td>
<td>$b = c / \tan C$</td>
</tr>
<tr>
<td></td>
<td>$B = 90^\circ - C$</td>
</tr>
</tbody>
</table>
### Table 2: Oblique Sided Triangle

<table>
<thead>
<tr>
<th>Known Sides &amp; Angles</th>
<th>Formulas for Unknown Sides and Angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 side &amp; 2 angles</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( b = \frac{a \cdot \sin B}{\sin A} \quad c = \frac{a \cdot \sin C}{\sin A} )</td>
</tr>
<tr>
<td>2 sides &amp; their connecting angle</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( B = 180^\circ - (A + C) \quad c = \sqrt{a^2 + b^2 - 2ab \cos C} )</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \tan A = \frac{a \cdot \sin C}{b - (a \cdot \cos C)} )</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>2 sides &amp; angle opposite one of the sides</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \sin B = \frac{b \cdot \sin A}{a} \quad c = \frac{a \cdot \sin C}{\sin A} \quad C = 180^\circ - (A + B) )</td>
</tr>
<tr>
<td>All 3 sides</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \cos A = \frac{b^2 + c^2 - a^2}{2bc} \quad \sin B = \frac{b \cdot \sin A}{a} \quad C = 180^\circ - (A + B) )</td>
</tr>
</tbody>
</table>

1. Area of an oblique or right sided triangle, ft.

\[
Area = \frac{a \cdot b \cdot \sin C}{2}
\]

\( a = \) Hypotenuse (Base), ft
\( b = \) Side, ft
\( C = \) Angle, deg
2. Area of an oblique or right sided triangle, \( \text{ft}\).

\[
Area = 0.5ah
\]
\(a = \) Hypotenuse (Base), \( \text{ft}\)
\(h = \) Height of triangle, \( \text{ft}\) (Eq #3)

3. Height of an oblique or right sided triangle, \( \text{ft}\).

\[
h = c \sin B
\]
\(c = \) Side, \( \text{ft}\)
\(B = \) Angle, \( \text{deg}\)

4. Perimeter of an oblique or right sided triangle, \( \text{ft}\).

\[
p = a + b + c
\]
\(a = \) Side, \( \text{ft}\)
\(b = \) Side, \( \text{ft}\)
\(c = \) Side, \( \text{ft}\)

**Turn / Swerve**

* Reference the Radius section for further variables.

**Linear Distance, Swerve**

1. Linear distance required with no braking at the commencement of a swerve to avoid a stationary hazard, \( \text{ft}\).

\[
d_x = V \sqrt{2d_y / a_y}
\]
\(V = \) Velocity, \( \text{ft/sec}\)
\(a_y = \) Lateral acceleration rate, \( \text{ft/sec}^2\)
\(d_y = \) Lateral avoidance distance, \( \text{ft}\)
2. Linear distance required with no braking at the commencement of a swerve to avoid a stationary hazard, \textbf{ft}.

\[ d_x = 0.366S \sqrt{d_y / f_y} \]

\( S = \text{Speed, mi/hr} \)
\( f_y = \text{Lateral acceleration factor, decimal} \)
\( d_y = \text{Lateral avoidance distance, ft} \)

G. Linear distance required with no braking at the commencement of a swerve to avoid a stationary hazard, \textbf{ft}.

\[ d_x = 0.225V \sqrt{d_y / f_y} \]

\( V = \text{Velocity, ft/sec} \)
\( f_y = \text{Lateral acceleration factor, decimal} \)
\( d_y = \text{Lateral avoidance distance, ft} \)

**Lateral Distance, Swerve**

3. Lateral distance in a swerve knowing the linear distance and speed, \textbf{ft}.

\[ d_y = 7.54 f_y (d_x / S)^2 \]

\( S = \text{Speed, mi/hr} \)
\( d_x = \text{Linear distance, ft} \)
\( f_y = \text{Lateral acceleration factor, decimal} \)

4. Distance traveled in a 90° turn knowing the radius traversed, \textbf{ft}.

\[ d_t = r \pi / 2 \]

\( r = \text{Radius traveled by center of mass, ft} \)
\( \pi = \text{Pi, } 3.141592654 \)

5. Turning radius during a swerve knowing initial speed and lateral acceleration factor, \textbf{ft}.

\[ r = S^2 / (15 f_y) \]

\( S = \text{Speed constant, mi/hr} \)
\( f_y = \text{Lateral acceleration factor, decimal} \)
6. Turning radius during a swerve knowing initial velocity and lateral acceleration factor, \( \text{ft} \).

\[
r = \frac{V^2}{f_y g}
\]

\( V = \) Velocity constant, \( \text{ft/sec} \)
\( f_y = \) Lateral acceleration factor, \( \text{decimal} \)
\( g = \) Gravitational constant, \( 32.2 \text{ ft/sec}^2 \)

7. Radius traveled by the vehicle's center of mass during a turn knowing the velocity and lateral acceleration rate, \( \text{ft} \).

\[
r = \frac{V^2}{a_y}
\]

\( V = \) Velocity constant, \( \text{ft/sec} \)
\( a_y = \) Lateral acceleration rate, \( \text{ft/sec}^2 \)

8. Radius traveled in a turn knowing the initial speed and lateral acceleration factor, \( \text{ft} \).

\[
r = \left( \frac{S}{3.86} \right)^2 \left( 1 - f_y e \right) / \left( f_y + e \right)
\]

\( S = \) Speed, \( \text{mi/hr} \)
\( f_y = \) Lateral acceleration factor, \( \text{decimal} \)
\( e = \) Superelevation of curve, \( \text{decimal} \)
\( \text{(negative value (-) for decline)} \)

9. Radius traveled during a turn knowing the wheelbase of the vehicle and the radians of an effective turn angle, \( \text{ft} \).

\[
r = \ell / \delta
\]

\( \ell = \) Wheelbase, \( \text{ft} \)
\( \delta = \) Effective turn angle to the front wheels, \( \text{rad} \) (Eq #11)

9. Radius traveled during a turn knowing the wheelbase of the vehicle and the degrees of an effective turn angle, \( \text{ft} \).

\[
r = \ell / \sin \delta_1
\]

\( \ell = \) Wheelbase, \( \text{ft} \)
\( \delta_1 = \) Effective turn angle to the front wheels, \( \text{deg} \) (Eq #13)
10. Maximum effective turn angle to the front wheels during a swerve or turn knowing the speed, wheelbase and lateral acceleration factor, rad.

\[ \delta = \frac{15\ell f_y}{S^2} \]

- \( S \) = Speed, mi/hr
- \( \ell \) = Wheelbase, ft
- \( f_y \) = Lateral acceleration factor, decimal

12. Radians of an effective turn angle to the front wheels knowing the degrees, rad.

\[ \delta = \delta_i \frac{\pi}{180} \]

- \( \delta_i \) = Effective turn angle to the front wheels, deg (Eq #14)
- \( \pi \) = Pi, 3.141592654

13. Degree of an effective turn angle to the front wheels knowing the radians, deg.

\[ \delta_i = \frac{\delta \pi}{180} \]

- \( \delta_i \) = Effective turn angle to the front wheels, rad (Eq #11)
- \( \pi \) = Pi, 3.141592654

14. Maximum effective turn angle to the front wheels during a swerve or turn knowing the radius and wheelbase, deg.

\[ \delta_i = \sin^{-1} \left( \frac{\ell}{r} \right) \]

- \( \ell \) = Wheelbase, ft
- \( r \) = Radius traveled by center of mass, ft

15. Total rotation to the steering wheel during a swerve or turn, deg.

\[ Sr = \delta_i \times fs \]

- \( \delta_i \) = Effective turn angle to the front wheels, deg (Eq #13, 14)
- \( fs \) = Steering ratio, 00:1
15. Lateral acceleration factor in a swerve knowing speed and radius traveled by the center of mass, \textit{decimal}.

\[ f_y = S^2 / (15r) \]
\[ S = \text{Speed constant, mi/hr} \]
\[ r = \text{Radius traveled by center of mass, ft} \]

16. Lateral acceleration factor in a swerve knowing the lateral and linear distances traveled at a known speed, \textit{decimal}.

\[ f_y = d_y 0.133 / \left(d_x / S\right)^2 \]
\[ S = \text{Speed constant, mi/hr} \]
\[ d_x = \text{Linear distance, ft} \]
\[ d_y = \text{Lateral avoidance distance, ft} \]

17. Lateral acceleration factor required in a turn to accelerate from one speed to another over a determined distance traveled in the turn, \textit{decimal}.

\[ f_y = \left(Sf^2 - So^2\right) / 30d_r \]
\[ Sf = \text{Speed final, mi/hr} \]
\[ So = \text{Speed initial, mi/hr} \]
\[ d_r = \text{Distance in turn, ft (Eq \#12, 13 Radius section)} \]

18. Lateral acceleration rate in a swerve knowing the velocity and radius traveled by the center of mass, \textit{ft/sec}².

\[ a_y = V^2 / r \]
\[ V = \text{Velocity constant, ft/sec} \]
\[ r = \text{Radius traveled by center of mass, ft} \]

19. Lateral acceleration rate required in a turn to accelerate from one velocity to another over a determined distance traveled in the turn, \textit{ft/sec}².

\[ a_y = \left(Vf^2 - Vo^2\right) / 2d_r \]
\[ Vf = \text{Velocity final, ft/sec} \]
\[ Vo = \text{Velocity initial, ft/sec} \]
\[ d_r = \text{Distance in turn, ft (Eq \#12, 13 Radius section)} \]
20. Speed in a swerve knowing the linear and lateral distances traversed and lateral acceleration factor, \textit{mi/hr}.

\[ S = d_x \times 2.73 / \sqrt{d_y / f_y} \]

- $f_y$ = Lateral acceleration factor, decimal
- $d_x$ = Linear distance, ft
- $d_y$ = Lateral avoidance distance, ft

22. Time in a swerve or turn knowing the velocity and distance traveled, \textit{sec}.

\[ T = d_t / V \]

- $V$ = Velocity constant, ft/sec
- $d_t$ = Distance in turn, ft (Eq #12, 13 Radius section)

23. Time required to accelerate/decelerate at the commencement of the turn from one velocity to another, \textit{sec}.

\[ T = (V_f - V_o) / a_y \]

- $V_f$ = Velocity final, ft/sec
- $V_o$ = Velocity initial, ft/sec
- $a_y$ = Lateral acceleration rate, ft/sec²
  
  (negative value (-) for deceleration)

24. Maximum velocity possible for a swerve with 100% braking knowing the lateral distance to avoid and lateral acceleration factor, \textit{ft/sec}.

\[ V = 15.50 \mu \sqrt{d_y / \mu} \]

- $\mu$ = Lateral Acceleration factor, decimal
- $d_y$ = Lateral distance to avoid, ft

\textit{Bonnett}

25. Maximum velocity possible for a swerve and return with 100% braking knowing the lateral distance to avoid and lateral acceleration factor, \textit{ft/sec}.

\[ V = 31.84 \mu \sqrt{d_y / 2 / \mu} \]

- $\mu$ = Lateral Acceleration factor, decimal
- $d_y$ = Lateral distance to avoid, ft

\textit{Bonnett}
Critical Speed Scuff

26. Velocity of a vehicle from a critical speed scuff with a known radius traveled at the center of mass, \textbf{ft/sec}. Superelevation is less than 6.8° (11.9 %).

\[ V = \sqrt{r(\mu + e)g} \]

- \( r \) = Radius traveled by center of mass, \text{ ft}
- \( \mu \) = Friction coefficient, \text{ decimal}
- \( e \) = Superelevation, maximum 11.9\%, \text{ decimal}
  (negative value (-) for decline)
- \( g \) = Gravitational constant, 32.2 ft/sec\(^2\)

27. Velocity of a vehicle from a critical speed scuff with an unknown radius traveled at the center of mass, \textbf{ft/sec}. Superelevation is less than 6.8° (11.9 %).

\[ V = \sqrt{(r - 0.5tw)(\mu + e)g} \]

- \( r \) = Radius of yaw mark, \text{ ft}
- \( tw \) = Track width, \text{ ft}
- \( \mu \) = Friction coefficient, \text{ decimal}
- \( e \) = Superelevation, maximum 11.9\%, \text{ decimal}
  (negative value (-) for decline)
- \( g \) = Gravitational constant, 32.2 ft/sec\(^2\)

28. Velocity of a vehicle from a critical speed scuff with an unknown radius traveled at the center of mass, \textbf{ft/sec}. Superelevation is either positive or negatively banked greater than 6.8° (11.9 %).

\[ V = \sqrt{\frac{(r - 0.5tw)(\mu + e)g}{1 - \mu e}} \]

- \( r \) = Radius of yaw mark, \text{ ft}
- \( tw \) = Track width, \text{ ft}
- \( \mu \) = Friction coefficient, \text{ decimal}
- \( e \) = Superelevation, minimum 11.9\%, \text{ decimal}
  (negative value (-) for decline)
- \( g \) = Gravitational constant, 32.2 ft/sec\(^2\)
29. Velocity of a vehicle from a critical speed scuff with a known side slip angle and radius traveled at the center of mass, \( \text{ft/sec} \).

\[
V = \sqrt{gr\left(\mu \cos \theta \cos \beta + \sin \theta \right)} / \left(\cos \theta - \mu \sin \theta \cos \beta \right)
\]

\( r \) = Radius traveled by center of mass, \( \text{ft} \)
\( \mu \) = Friction coefficient, decimal
\( \beta \) = Side slip angle, \( \text{deg} \) (angle of centrifugal skid striations)
\( \theta \) = Grade in direction of side slip, \( \text{deg} \) (negative value (-) for decline)
\( g \) = Gravitational constant, 32.2 \( \text{ft/sec}^2 \)

---

g. Velocity of a vehicle from a critical speed scuff with a known side slip angle, radius traveled at the center of mass, and longitudinal acceleration, \( \text{ft/sec} \).

\[
V = \sqrt{gr\left[\left(f_x \cos \beta - f_x \sin \beta\right)\left(\sin \theta \tan \theta + \cos \theta\right) + \tan \theta\right]} \quad \text{positive grade}
\]

\[
V = \sqrt{gr\left[\left(f_x \cos \beta - f_x \sin \beta\right)\left(\cos \theta - \sin \theta \tan \theta\right) + \tan \theta\right]} \quad \text{negative grade}
\]

\( f_y = \sqrt{\mu^2 - f_x^2} \)

\( r \) = Radius traveled by center of mass, \( \text{ft} \)
\( \mu \) = Friction Coefficient, decimal
\( f_x \) = Longitudinal Friction Coefficient, decimal
\( f_y \) = Lateral Friction Coefficient, decimal
\( \beta \) = Side slip angle, \( \text{deg} \) (angle of centrifugal skid striations)
\( \theta \) = Grade in direction of side slip, \( \text{deg} \)
\( g \) = Gravitational constant, 32.2 \( \text{ft/sec}^2 \)
30. Final velocity of an acceleration/deceleration in a turn knowing the initial velocity, lateral acceleration rate and distance traveled in the turn, \( \text{ft/sec} \).

\[
V_f = \sqrt{V_0^2 + 2a_yd}
\]

- \( V_0 \) = Velocity initial, \( \text{ft/sec} \)
- \( a_y \) = Lateral acceleration rate, \( \text{ft/sec}^2 \)
- \( d \) = Distance in turn, \( \text{ft} \) (Eq #12, 13 Radius section)
- (negative value (-) for deceleration)

31. Cornering force applied to a tire, which is at its frictional limit, \( \text{lb} \).

\[
F_y = W\mu\cos\alpha
\]

- \( \mu \) = Friction coefficient, decimal
- \( \alpha \) = Tire slip angle, \( \text{deg} \)
- \( W \) = Weight on tire, \( \text{lb} \)

32. Braking force applied to a tire, which is at its frictional limit, \( \text{lb} \).

\[
F_{sb} = W\mu\sin\alpha
\]

- \( \mu \) = Friction coefficient, decimal
- \( \alpha \) = Tire slip angle, \( \text{deg} \)
- \( W \) = Weight on tire, \( \text{lb} \)

33. Yaw mark authentication. If the solution is between 0.10 and 0.20 for a deceleration factor, then the mark is the result of an accurate yaw with no braking applied, \( \text{decimal} \).

\[
Y_m = \frac{0.033(S_1^2 - S_2^2)}{d}
\]

- \( S_1 \) = First section of yaw utilized to determine the speed, \( \text{mi/hr} \)
- \( S_2 \) = Second section of yaw measured from the end of the first chord length, \( \text{mi/hr} \)
- \( d \) = Distance between the two middle ordinates, \( \text{ft} \)
**Velocity**

A vector quantity whose magnitude is a body's speed and whose direction is the body's direction of motion. The rate of speed of action or occurrence.

1. Constant velocity over a determined distance and a unit of time, **ft/sec**.

   \[ V = \frac{d}{T} \]
   
   \[ d = \text{Distance, ft} \]
   
   \[ T = \text{Time, sec} \]

2. Equivalent constant velocity from a known speed, **ft/sec**.

   \[ V = 1.466S \]
   
   \[ S = \text{Speed constant, mi/hr} \]

3. Equivalent constant velocity from a known speed, **ft/sec**.

   \[ V = \frac{S}{60^2/5280} \]
   
   \[ S = \text{Speed constant, mi/hr} \]

4. Velocity gained or lost after accelerating/decelerating over a unit of time, **ft/sec**.

   \[ V = aT \]
   
   \[ a = \text{Accel / Decel rate, ft/sec}^2 \]
   
   \[ T = \text{Time, sec} \]

5. Velocity gained or lost after accelerating/decelerating over a unit of time, **ft/sec**.

   \[ V = fgT \]
   
   \[ f = \text{Accel / Decel factor, decimal} \]
   
   \[ T = \text{Time, sec} \]
   
   \[ g = \text{Gravitational constant, 32.2 ft/sec}^2 \]

6. Final velocity after accelerating/decelerating over a unit of time, **ft/sec**.

   \[ V_f = V_0 + aT \]
   
   \[ V_0 = \text{Velocity initial, ft/sec} \]
   
   \[ a = \text{Accel / Decel rate, ft/sec}^2 \]
   
   (negative value (-) for deceleration)
   
   \[ T = \text{Time, sec} \]
7. Final velocity after accelerating/decelerating over a unit of time, \( \text{ft/sec} \).

\[
V_f = Vo + fgT
\]

\( Vo \) = Velocity initial, \( \text{ft/sec} \)
\( f \) = Accel / Decel factor, decimal
\( g \) = Gravitational constant, 32.2 \( \text{ft/sec}^2 \)
\( T \) = Time, sec

8. Initial velocity after accelerating/decelerating over a unit of time, \( \text{ft/sec} \).

\[
Vo = Vf - aT
\]

\( Vf \) = Velocity final, \( \text{ft/sec} \)
\( a \) = Accel / Decel rate, \( \text{ft/sec}^2 \)
\( T \) = Time, sec

9. Initial velocity after accelerating/decelerating over a unit of time, \( \text{ft/sec} \).

\[
Vo = Vf - fgT
\]

\( Vf \) = Velocity final, \( \text{ft/sec} \)
\( f \) = Accel / Decel factor, decimal
\( g \) = Gravitational constant, 32.2 \( \text{ft/sec}^2 \)
\( T \) = Time, sec

10. Velocity of an acceleration/deceleration from or to a stop over a determined distance and a unit of time, \( \text{ft/sec} \).

\[
V = 2d / T
\]

\( d \) = Distance, \( \text{ft} \)
\( T \) = Time, sec

11. Velocity of an acceleration/deceleration from or to a stop, \( \text{ft/sec} \).

\[
V = \sqrt{2.fgd}
\]

\( f \) = Accel / Decel factor, decimal
\( d \) = Distance, \( \text{ft} \)
\( g \) = Gravitational constant, 32.2 \( \text{ft/sec}^2 \)
12. Velocity of an acceleration/deceleration from or to a stop, **ft/sec**.

\[ V = \sqrt{2ad} \]

\[ a = \text{Accel / Decel rate, ft/sec}^2 \]
\[ d = \text{Distance, ft} \]

13. Initial velocity of a deceleration to a stop on a surface grade less than 6.8° (11.9 %), **ft/sec**.

\[ V = \sqrt{2gd(\mu n \pm m)} \]

\[ d = \text{Distance, ft} \]

\[ \mu = \text{Level friction coefficient, decimal} \]
\[ g = \text{Gravitational constant, 32.2 ft/sec}^2 \]
\[ m = \text{Grade, maximum 11.9%, decimal} \]
\[ n = \text{Braking efficiency, decimal} \]

14. Final velocity of an accel/deceleration over a determined distance, **ft/sec**.

\[ Vf = \sqrt{Vo^2 + 2ad} \]

\[ Vo = \text{Velocity initial, ft/sec} \]
\[ a = \text{Accel / Decel rate, ft/sec}^2 \]
\[ (\text{negative value (-) for deceleration}) \]
\[ d = \text{Distance, ft} \]

15. Final velocity of an accel/deceleration over a determined distance, **ft/sec**.

\[ Vf = \sqrt{Vo^2 + 2fgd} \]

\[ Vo = \text{Velocity initial, ft/sec} \]
\[ f = \text{Accel / Decel factor, decimal} \]
\[ (\text{negative value (-) for deceleration}) \]
\[ d = \text{Distance, ft} \]
\[ g = \text{Gravitational constant, 32.2 ft/sec}^2 \]

16. Initial velocity of an accel/deceleration over a determined distance, **ft/sec**.

\[ Vo = \sqrt{Vf^2 - 2ad} \]

\[ Vf = \text{Velocity final, ft/sec} \]
\[ a = \text{Accel / Decel rate, ft/sec}^2 \]
\[ (\text{negative value (-) for deceleration}) \]
\[ d = \text{Distance, ft} \]
17. Initial velocity of an accel/deceleration over a determined distance, $\text{ft/sec}$.

\[ V_o = \sqrt{V_f^2 - 2fgd} \]  
\[ V_f = \text{Velocity final, \ ft/sec} \]  
\[ f = \text{Accel / Decel factor, \ decimal} \]  
\[ (\text{negative value (-) for deceleration}) \]  
\[ d = \text{Distance, \ ft} \]  
\[ g = \text{Gravitational constant, \ 32.2 \ ft/sec}^2 \]  

18. Combined velocity of a deceleration over several surfaces, $\text{ft/sec}$.

\[ V = \sqrt{V_1^2 + V_2^2 + V_3^2 + \ldots + V_n^2} \]  
\[ V_1 \rightarrow V_n = \text{Pre-determined velocities of individual surfaces, \ ft/sec} \]  

19. Combined velocity of a deceleration over two surfaces, $\text{ft/sec}$.

\[ V = \sqrt{V_o^2 + 2ad} \]  
\[ V_o = \text{Velocity from 1st surface, \ ft/sec} \]  
\[ a = \text{Deceleration rate, \ ft/sec}^2 \]  
\[ d = \text{Distance (2nd surface), \ ft} \]  

20. Initial velocity of a deceleration over a determined distance and a unit of time, $\text{ft/sec}$.

\[ V_o = \left( d + \left( a / 2 \right) T^2 \right) / T \]  
\[ d = \text{Distance, \ ft} \]  
\[ T = \text{Time, \ sec} \]  
\[ a = \text{Deceleration rate, \ ft/sec}^2 \]  

21. Final velocity of a deceleration over a determined distance and a unit of time, $\text{ft/sec}$.

\[ V_f = \left( d - \left( a / 2 \right) T^2 \right) / T \]  
\[ d = \text{Distance, \ ft} \]  
\[ T = \text{Time, \ sec} \]  
\[ a = \text{Deceleration rate, \ ft/sec}^2 \]
22. Final velocity of an acceleration over a determined distance and a unit of time, \( \text{ft/sec} \).

\[ V_f = \frac{d}{T} + \frac{aT}{2} \]

- \( d \) = Distance, ft
- \( T \) = Time, sec
- \( a \) = Acceleration rate, \( \text{ft/sec}^2 \)

23. Initial velocity of an acceleration over a determined distance and a unit of time, \( \text{ft/sec} \).

\[ V_i = \frac{d}{T} - \frac{aT}{2} \]

- \( d \) = Distance, ft
- \( T \) = Time, sec
- \( a \) = Acceleration rate, \( \text{ft/sec}^2 \)

**Brake Lag**

24. Velocity at commencement of brake activation incorporating the time of brake lag, \( \text{ft/sec} \).

\[ V_b = V + 0.6aT_b \]

- \( V \) = Initial velocity calculated, \( \text{ft/sec} \)
- \( a \) = Deceleration rate, \( \text{ft/sec}^2 \)
- \( T_b \) = Brake lag time, sec

_Eubanks_

Recommended brake lag time of 0.3 - 0.55 seconds for standard brake systems.

**Kinetic Energy**

25. Determine a velocity knowing the kinetic energy generated and weight of the object, \( \text{ft/sec} \).

\[ V = \sqrt{\frac{2E}{W}} \]

- \( E \) = Kinetic energy, \( \text{ft-lb} \)
- \( W \) = Total static weight, lb
- \( g \) = Gravitational constant, 32.2 \( \text{ft/sec}^2 \)

26. Velocity of a vehicle from a critical speed scuff with a known radius traveled at the center of
mass, **ft/sec**. Superelevation is less than 6.8° (11.9 %).

\[
V = \sqrt{r(\mu + e)g}
\]

- \( r \) = Radius traveled by center of mass, **ft**
- \( \mu \) = Friction Coefficient, decimal
- \( e \) = Superelevation, maximum 11.9%, decimal
  {(-) value for negative banking}
- \( g \) = Gravitational constant, 32.2 **ft/sec**²

27. Velocity of a vehicle from a critical speed scuff with an unknown radius traveled at the center of mass, **ft/sec**. Superelevation is less than 6.8° (11.9 %).

\[
V = \sqrt{(r - 0.5tw)(\mu + e)g}
\]

- \( r \) = Radius of yaw mark, **ft**
- \( tw \) = Track width, **ft**
- \( \mu \) = Friction Coefficient, decimal
- \( e \) = Superelevation, maximum 11.9%, decimal
  {(-) value for negative banking}
- \( g \) = Gravitational constant, 32.2 **ft/sec**²

28. Velocity of a vehicle from a critical speed scuff with an unknown radius traveled at the center of mass, **ft/sec**. Superelevation is either positive or negatively banked greater than 6.8° (11.9 %).

\[
V = \sqrt{\frac{(r - 0.5tw)(\mu + e)g}{1 - \mu e}}
\]

- \( r \) = Radius of yaw mark, **ft**
- \( tw \) = Track width, **ft**
- \( \mu \) = Friction Coefficient, decimal
- \( e \) = Superelevation, minimum 11.9 %, decimal
  (negative value (-) for decline)
- \( g \) = Gravitational constant, 32.2 **ft/sec**²

29. Velocity of a vehicle from a critical speed scuff with a known side slip angle and radius traveled
at the center of mass, \textbf{ft/sec}. \\
\[ V = \sqrt{gr\left(\mu \cos \theta \cos \beta + \sin \theta\right)}/\left(\cos \theta - \mu \sin \theta \cos \beta\right) \]

\[ r = \text{Radius traveled by center of mass, ft} \]
\[ \mu = \text{Friction Coefficient, decimal} \]
\[ \beta = \text{Side slip angle, deg (angle of centrifugal skid striations)} \]
\[ \theta = \text{Grade in direction of side slip, deg (negative value (-) for decline)} \]
\[ g = \text{Gravitational constant, 32.2 ft/sec}^2 \]

30. Velocity of a vehicle from a critical speed scuff with a known side slip angle, radius traveled at the center of mass, and longitudinal acceleration, \textbf{ft/sec}. \\
\[ V = \sqrt{gr\left[\left(f_y \cos \beta - f_x \sin \beta\right)(\sin \theta \tan \theta + \cos \theta) + \tan \theta\right]} \textbf{ positive grade} \]
\[ V = \sqrt{gr\left[\left(f_y \cos \beta - f_x \sin \beta\right)(\cos \theta - \sin \theta \tan \theta) + \tan \theta\right]} \textbf{ negative grade} \]

\[ f_y = \sqrt{\mu^2 - f_x^2} \]

\[ r = \text{Radius traveled by center of mass, ft} \]
\[ \mu = \text{Friction Coefficient, decimal} \]

Northwestern
\[ f_x = \text{Longitudinal Friction Coefficient, decimal} \]
\[ f_y = \text{Lateral Friction Coefficient, decimal} \]
\[ \beta = \text{Side slip angle, deg (angle of centrifugal skid striations)} \]
\[ \theta = \text{Grade in direction of side slip, deg} \]
\[ g = \text{Gravitational constant, 32.2 ft/sec}^2 \]

31. Velocity accounting for weight shift during a deceleration, \textbf{ft/sec}. 

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\[ V = \sqrt{2g \left[ f_F d_F \left( W_F + \Delta W \right) + f_R d_R \left( W_R - \Delta W \right) \right]} \]

- \( d \) = Total distance center of mass traveled, ft
- \( d_F \) = Front axle skid distance, ft
- \( d_R \) = Rear axle skid distance, ft
- \( f_F \) = Front deceleration factor, decimal
- \( f_R \) = Rear deceleration factor, decimal
- \( W \) = Total static weight, lb
- \( W_F \) = Static weight of front axle, lb
- \( W_R \) = Static weight of rear axle, lb
- \( \Delta W \) = Weight shift to front axle, lb

(Eq #1, 2, 3 Weight Shift section)
- \( e \) = Superelevation, maximum 11.9\%, decimal
  (negative value (-) for decline)
- \( g \) = Gravitational constant, \( 32.2 \text{ ft/sec}^2 \)

**Skip Skid Marks**

32. Velocity from the measurements of skip skid marks, \( \text{ft/sec} \). The road surface must be even and level.

\[
V = \sqrt{\frac{161 f (n-2)^2 d_o^2}{(n-2)d_o - d_T}}
\]

- \( f \) = Deceleration factor, decimal
- \( n \) = Number of skip skid marks, 
- \( d_o \) = Distance between the first pair of marks, ft
- \( d_T \) = Distance between the middle of the first and last pairs of marks, ft

**Kwasnoski**

33. Velocity calculated for a vehicle sliding laterally in an original forward movement due to roadway
grade, **ft/sec**.

\[ V = 2f \sqrt{\frac{gd_y}{\sin \alpha}} \]

\[ f = \text{Deceleration factor, decimal} \]

\[ d_y = \text{Lateral distance, ft} \]

**Limpert**

\[ \alpha = \text{Lateral or cross slope angle, deg (10 degrees or less)} \]

\[ g = \text{Gravitational constant, 32.2 ft/sec}^2 \]

34. Longitudinal distance vehicle will travel while sliding laterally from Eq #33, **ft**.

\[ d_s = \frac{2fd_y}{\sin \alpha} \]

\[ d_s = \text{Lateral sliding distance, ft} \]

**Limpert**

\[ \alpha = \text{Cross slope angle, deg} \]

\[ f = \text{Friction coefficient, decimal} \]

**Weight Shift**

A transfer of weight from one portion of a vehicle to another as a consequence of an acceleration or deceleration. Caused by the redistribution of external forces that must occur when a body of finite size is accelerated or decelerated by non centroidal forces.

1. Weight shifted from/to the front axle during an accel/deceleration, **lb**. Basic equation.

\[ \Delta W = \frac{Wfz}{\ell} \]

\[ f = \text{Accel / Decel factor, decimal} \]

\[ z = \text{Vertical center of mass height, ft} \]

(Eq #7, 8 Center of Mass section)

\[ \ell = \text{Wheelbase, ft} \]

\[ W = \text{Total static weight, lb} \]

2. Weight shifted to the front axle during a deceleration, **lb**. Basic equation.
\[
\Delta W = \frac{Fz}{\ell}
\]

\(F = \text{Force, lb (Eq #1, 2, 3 Force section)}\)
\(z = \text{Vertical center of mass height, ft (Eq #7, 8 Center of Mass section)}\)
\(\ell = \text{Wheelbase, ft}\)

3. Weight shifted from/to the front axle during an accel/deceleration, \(\text{lb}\).

\[
\Delta W = \frac{z / \ell (W_F f_F + W_R f_R)}{(1 - z / \ell f_F) + z / \ell f_R}
\]

\(f_F = \text{Accel / Decel factor of front axle, decimal}\)
\(f_R = \text{Accel / Decel factor of rear axle, decimal}\)
\(z = \text{Vertical center of mass height, ft}\)
\(\ell = \text{Wheelbase, ft}\)
\(W_F = \text{Static weight on front axle, lb}\)
\(W_R = \text{Static weight on rear axle, lb}\)

4. Weight shifted to the outside tires during corning, \(\text{lb}\).

\[
\Delta W_o = \frac{z / tw (W_o f_o + W_i f_i)}{(1 - z / tw f_o) + z / tw f_i}
\]

\(f_o = \text{Lateral acceleration factor of outside tires, decimal}\)
\(f_i = \text{Lateral acceleration factor of inside tires, decimal}\)
\(z = \text{Vertical center of mass height, ft}\)
\(tw = \text{Track width, ft}\)
\(W_o = \text{Static weight on outside tires, lb}\)
\(W_i = \text{Static weight on inside tires, lb}\)

5. Weight shifted from one side of a vehicle to the other during cornering, \(\text{lb}\).
\[ \Delta W_y = \left( W_{a_y} z \right) / \left( g \cdot 2 \cdot y \right) \]

\( a_y \) = Lateral acceleration rate, \( \text{ft/} \text{sec}^2 \)

\( y \) = Lateral center of mass location, \( \text{ft} \)  
(Eq #5, 6 Center of Mass section)  

\( z \) = Vertical center of mass height, \( \text{ft} \)  
(Eq #7, 8 Center of Mass section)  

\( W \) = Total static weight, \( \text{lb} \)  

\( g \) = Gravitational constant, \( 32.2 \text{ ft/} \text{sec}^2 \)

6. Weight on the inside, front-tire during cornering, \( \text{lb} \).

\[ W_{fi} = W_F / 2 - \left( W_{a_y} z \right) / \left( 4gy \right) \]

\( a_y \) = Lateral acceleration rate, \( \text{ft/} \text{sec}^2 \)

\( y \) = Lateral center of mass location, \( \text{ft} \)  
(Eq #5, 6 Center of Mass section)  

\( z \) = Vertical center of mass height, \( \text{ft} \)  
(Eq #7, 8 Center of Mass section)  

\( W_F \) = Static front axle weight, \( \text{lb} \)

\( W \) = Total static weight, \( \text{lb} \)

\( g \) = Gravitational constant, \( 32.2 \text{ ft/} \text{sec}^2 \)

7. Weight on the outside, front-tire during cornering, \( \text{lb} \).

\[ W_{fo} = W_F / 2 + \left( W_{a_y} z \right) / \left( 4gy \right) \]

\( a_y \) = Lateral acceleration rate, \( \text{ft/} \text{sec}^2 \)

\( y \) = Lateral center of mass location, \( \text{ft} \)  
(Eq #5, 6 Center of Mass section)  

\( z \) = Vertical center of mass height, \( \text{ft} \)  
(Eq #7, 8 Center of Mass section)  

\( W_F \) = Static front axle weight, \( \text{lb} \)

\( W \) = Total static weight, \( \text{lb} \)

\( g \) = Gravitational constant, \( 32.2 \text{ ft/} \text{sec}^2 \)

8. Weight on the inside, rear tire during cornering, \( \text{lb} \).
\[ W_{Ri} = \frac{W_R}{2} - \left( \frac{W_R}{a_y} \right) / (4gy) \]

\[ a_y = \text{Lateral acceleration rate, ft/sec}^2 \]

\[ y = \text{Lateral center of mass location, ft} \]

(Eq #5, 6 Center of Mass section)

\[ z = \text{Vertical center of mass height, ft} \]

(Eq #7, 8 Center of Mass section)

\[ W_R = \text{Static rear axle weight, lb} \]

\[ W = \text{Total static weight, lb} \]

\[ g = \text{Gravitational constant, 32.2 ft/sec}^2 \]

9. Weight on the outside, rear tire during cornering, \textbf{lb}.

\[ W_{Ro} = \frac{W_R}{2} + \left( \frac{W_R}{a_y} \right) / (4gy) \]

\[ a_y = \text{Lateral acceleration rate, ft/sec}^2 \]

\[ y = \text{Lateral center of mass location, ft} \]

(Eq #5, 6 Center of Mass section)

\[ z = \text{Vertical center of mass height, ft} \]

(Eq #7, 8 Center of Mass section)

\[ W_R = \text{Static rear axle weight, lb} \]

\[ W = \text{Total static weight, lb} \]

\[ g = \text{Gravitational constant, 32.2 ft/sec}^2 \]

10. Weight on each front-tire during a linear acceleration/deceleration, \textbf{lb}.

\[ W_{Fx} = (W_F - W_{a_z}z_i / g) / 2 \]

\[ a_x = \text{Linear acceleration rate, ft/sec}^2 \]

(negative value (-) for deceleration)

\[ z_i = \text{Vertical center of mass height as a decimal fraction of the wheelbase (z/\ell), ft} \]

(Eq #9 Center of Mass section)

\[ W_F = \text{Static front axle weight, lb} \]

\[ W = \text{Total static weight, lb} \]

\[ g = \text{Gravitational constant, 32.2 ft/sec}^2 \]

11. Weight on each rear tire during a linear acceleration/deceleration, \textbf{lb}.
\[ W_{Rx} = \left( W_R + W a_x z_i / g \right) / 2 \]

\[ a_x = \text{Linear acceleration rate, ft/sec}^2 \]

(negative value (-) for deceleration)

\[ z_i = \text{Vertical center of mass height as a decimal fraction of the wheelbase (z/\ell), ft} \]

(Eq #9 Center of Mass section)

\[ W_F = \text{Static front axle weight, lb} \]

\[ W = \text{Total static weight, lb} \]

\[ g = \text{Gravitational constant, 32.2 ft/sec}^2 \]

12. Speed accounting for weight shift during a deceleration, mi/hr.

\[ S = \sqrt{\frac{30 \left[ f_F d_F (W_F + \Delta W) + f_R d_R (W_R - \Delta W) \right]}{W} + dm} \]

\[ d = \text{Total distance center of mass traveled, ft} \]

\[ d_F = \text{Front axle skid distance, ft} \]

\[ d_R = \text{Rear axle skid distance, ft} \]

\[ f_F = \text{Front deceleration factor, decimal} \]

\[ f_R = \text{Rear deceleration factor, decimal} \]

\[ W = \text{Total static weight, lb} \]

\[ W_F = \text{Static weight of front axle, lb} \]

\[ W_R = \text{Static weight of rear axle, lb} \]

\[ \Delta W = \text{Weight shift to front axle, lb} \] (Eq #1, 2, 3)

\[ m = \text{Grade, maximum 11.9\%, decimal} \]

(negative value (-) for decline)

\[ V = \sqrt{2g \left[ \frac{f_F d_F (W_F + \Delta W) + f_R d_R (W_R - \Delta W)}{W} \right] + dm} \]

\( d = \) Total distance center of mass traveled, ft
\( d_F = \) Front axle skid distance, ft
\( d_R = \) Rear axle skid distance, ft
\( f_F = \) Front deceleration factor, decimal
\( f_R = \) Rear deceleration factor, decimal
\( W = \) Total static weight, lb
\( W_F = \) Static weight of front axle, lb
\( W_R = \) Static weight of rear axle, lb
\( \Delta W = \) Weight shift to front axle, lb (Eq #1, 2, 3)
\( m = \) Grade, maximum 11.9%, decimal
  (negative value (-) for decline)
\( g = \) Gravitational constant, 32.2 ft/sec^2

14. Speed accounting for weight shift during a deceleration for a non-articulated vehicle with axle pairs
close together, **mi/hr.**

\[
S = \sqrt{\frac{f_{Fr}d_{Fr}(W_{Fr} + \Delta W / 2) + f_{Fl}d_{Fl}(W_{Fl} + \Delta W / 2) + f_{Rr}d_{Rr}(W_{Rr} - \Delta W / 2) + f_{Rl}d_{Rl}(W_{Rl} - \Delta W / 2)}{W}}
\]

- \(d_{Fr}\) = Right front-tire skid distance, ft
- \(d_{Fl}\) = Left front-tire skid distance, ft
- \(d_{Rr}\) = Right rear tire skid distance, ft
- \(d_{Rl}\) = Left rear tire skid distance, ft
- \(f_{Fr}\) = Right front deceleration factor, decimal
- \(f_{Fl}\) = Left front deceleration factor, decimal
- \(f_{Rr}\) = Right rear deceleration factor, decimal
- \(f_{Rl}\) = Left rear deceleration factor, decimal
- \(W\) = Total static weight, lb
- \(W_{Fr}\) = Static right front axle weight, lb
- \(W_{Fl}\) = Static left front axle weight, lb
- \(W_{Rr}\) = Static right rear axle weight, lb
- \(W_{Rl}\) = Static left rear axle weight, lb
- \(\Delta W\) = Weight shift to front axle, lb (Eq 1, 2, 3)

15. Velocity accounting for weight shift during a deceleration on a non-articulated vehicle with axle pairs close together, **ft/sec.**
\[ V = \sqrt{2g \left[ \frac{f_{Fr}d_{Fr} \left(W_{Fr} + \Delta W / 2\right) + f_{Fl}d_{Fl} \left(W_{Fl} + \Delta W / 2\right) + f_{Rr}d_{Rr} \left(W_{Rr} - \Delta W / 2\right) + f_{Rl}d_{Rl} \left(W_{Rl} - \Delta W / 2\right)}{W} \right]} \]

- \(d_{Fr}\) = Right front-tire skid distance, ft
- \(d_{Fl}\) = Left front-tire skid distance, ft
- \(d_{Rr}\) = Right rear-tire skid distance, ft
- \(d_{Rl}\) = Left rear-tire skid distance, ft
- \(f_{Fr}\) = Right-front deceleration factor, decimal
- \(f_{Fl}\) = Left-front deceleration factor, decimal
- \(f_{Rr}\) = Right-rear deceleration factor, decimal
- \(f_{Rl}\) = Left-rear deceleration factor, decimal
- \(W\) = Total static weight, lb
- \(W_{Fr}\) = Static right-front axle weight, lb
- \(W_{Fl}\) = Static left-front axle weight, lb
- \(W_{Rr}\) = Static right-rear axle weight, lb
- \(W_{Rl}\) = Static left-rear axle weight, lb
- \(\Delta W\) = Weight shift to front axle, lb (Eq #1, 2, 3)
- \(g\) = Gravitational constant, 32.2 ft/sec\(^2\)

**Weight**

The weight of a body is the gravitational force exerted on it by the earth. Weight, being a force, is a vector quantity. The direction of this vector is the direction of the gravitational force, which is toward the center of the earth. The magnitude of the weight is expressed in force units, such as pounds.

1. Weight of an object during an acceleration/deceleration knowing the force applied, lb.

\[ W = \frac{F}{f} \]

\(F\) = Force, lb (Eq #3 Force section)
\(f\) = Accel / Decel factor, decimal

2. Weight of an object from its mass and the acceleration of gravity, lb.
\[ W = mg \]

- \( W \) = Mass, \( \text{lb sec}^2 / \text{ft} \)
- \( m \) = Mass, \( \text{lb sec}^2 / \text{ft} \)
- \( g \) = Gravitational constant, 32.2 \( \text{ft/sec}^2 \)

3. Weight of an object during an accel/deceleration from one velocity to another knowing the kinetic energy generated or dissipated, \( \text{lb} \).

\[ W = 2E \left( \frac{(Vf^2 - Vo^2)}{g} \right) \]

- \( W \) = Weight of an object, \( \text{lb} \)
- \( E \) = Kinetic energy, \( \text{ft-lb} \) (Eq #7 Energy section)
  
  (negative value (-) for deceleration)
- \( Vf \) = Velocity final, \( \text{ft/sec} \)
- \( Vo \) = Velocity initial, \( \text{ft/sec} \)
- \( g \) = Gravitational constant, 32.2 \( \text{ft/sec}^2 \)

G. Load distribution on the front axle during an acceleration, \( \text{lb} \).

\[ W_f = \left( W \cdot x_R \cdot \cos \theta - F_xZ_h - V_zx_h - \frac{W}{g} a_xz - D_a h_a - Wz \cdot \sin \theta \right) / \ell \]

- \( W \) = Weight of vehicle, \( \text{lb} \)
- \( W_f \) = Load distribution on the front axle, \( \text{lb} \)
- \( x_R \) = Longitudinal center of mass from the rear axle, \( \text{ft} \) (Eq #3 Center of Mass section)
- \( \theta \) = Grade angle, \( \text{deg} \)
- \( F_x \) = Longitudinal forces acting at the hitch point when the vehicle is towing a trailer, \( \text{lb} \)
- \( Z_h \) = Center of hitch vertical height, \( \text{ft} \)
- \( V_z \) = Vertical load acting at the hitch point when the vehicle is towing a trailer, \( \text{lb} \)
- \( a_x \) = Acceleration in the \( x \)-direction, \( \text{ft/sec}^2 \)
- \( z \) = Vertical center of mass height, \( \text{ft} \)
- \( D_a \) = Aerodynamic drag force, \( \text{decimal} \)
- \( h_a \) = Height of the aerodynamic drag force, \( \text{ft} \)
- \( \ell \) = Wheelbase, \( \text{ft} \)
- \( g \) = Gravitational constant, 32.2 \( \text{ft/sec}^2 \)

G. Load distribution on the rear axle during an acceleration, \( \text{lb} \).
\[ W_F = \left( W_x F \cos \theta - F_z Z_h - V_z (x_h + \ell) + \frac{W a_x z - D_A h_a + W z \sin \theta}{g} \right) / \ell \]

**Gillespie**

- \( W = \) Weight of vehicle, lb
- \( x_F = \) Longitudinal center of mass from the front axle, ft (Eq #1 Center of Mass section)
- \( \theta = \) Grade angle, deg
- \( F_x = \) Longitudinal forces acting at the hitch point when the vehicle is towing a trailer, lb
- \( Z_h = \) Center of hitch vertical height, ft
- \( V_z = \) Vertical load acting at the hitch point when the vehicle is towing a trailer, lb
- \( x_h = \) Distance from rear axle to the hitch point, ft
- \( a_x = \) Acceleration in the \( x\)-direction, ft/sec\(^2\)
- \( z = \) Vertical center of mass height, ft
- \( D_A = \) Aerodynamic drag force, decimal
- \( h_a = \) Height of the aerodynamic drag force, ft
- \( \ell = \) Wheelbase, ft
- \( g = \) Gravitational constant, 32.2 ft/sec\(^2\)

**G.** \( W_F = W x_R / \ell \)

Static load for front axle on level ground, lb.

- \( \ell = \) Wheelbase, ft
- \( W = \) Total static Weight, lb
- \( x_R = \) Longitudinal center of mass from the rear axle, ft (Eq #3 Center of Mass section)

**G.** Static load for rear axle on level ground, lb.
\[ W_R = W x_F / \ell \]
\[ \ell = \text{Wheelbase, ft} \]
\[ W = \text{Total static Weight, lb} \]
\[ x_F = \text{Longitudinal center of mass from the front axle, ft} \]
(Eq #3 Center of Mass section)

N. Front axle weight during low speed acceleration, \( \text{lb} \).

\[ W_F = W \left( \frac{x_R}{\ell} - \frac{a_x z}{g \ell} \right) \]
\[ W = \text{Weight of vehicle, lb} \]
\[ \ell = \text{Wheelbase, ft} \]
\[ x_R = \text{Longitudinal center of mass from the rear axle, ft} \]
\[ a_x = \text{Linear acceleration rate, ft/sec}^2 \]
\[ z = \text{Vertical center of mass height, ft} \]
(Eq #7, 8 Center of Mass section)
\[ g = \text{Gravitational constant, 32.2 ft/sec}^2 \]

Gillespie

N. Rear axle weight during low speed acceleration, \( \text{lb} \).

\[ W_R = W \left( \frac{x_F}{\ell} + \frac{a_x z}{g \ell} \right) \]
\[ W = \text{Weight of vehicle, lb} \]
\[ \ell = \text{Wheelbase, ft} \]
\[ x_F = \text{Longitudinal center of mass from the front axle, ft} \]
\[ a_x = \text{Linear acceleration rate, ft/sec}^2 \]
\[ z = \text{Vertical center of mass height, ft} \]
(Eq #7, 8 Center of Mass section)
\[ g = \text{Gravitational constant, 32.2 ft/sec}^2 \]

Gillespie

N. Front axle weight during low speed acceleration on a grade, \( \text{lb} \).
\[ W_R = W \left( \frac{x_R}{\ell} - \frac{z}{\ell} \theta \right) \]

\[ W = \text{Weight of vehicle, lb} \]
\[ \ell = \text{Wheelbase, ft} \]
\[ x_R = \text{Longitudinal center of mass from the rear axle, ft} \]
\[ z = \text{Vertical center of mass height, ft} \]

(Eq #7, 8 Center of Mass section)
\[ \theta = \text{Grade angle, deg} \]

N. Rear axle weight during low speed acceleration on a grade, lb.

\[ W_R = W \left( \frac{x_F}{\ell} + \frac{z}{\ell} \theta \right) \]

\[ W = \text{Weight of vehicle, lb} \]
\[ \ell = \text{Wheelbase, ft} \]
\[ x_F = \text{Longitudinal center of mass from the front axle, ft} \]
\[ z = \text{Vertical center of mass height, ft} \]

(Eq #7, 8 Center of Mass section)
\[ \theta = \text{Grade angle, deg} \]
## Reference Data

### Acceleration / Deceleration Factor

#### Table 1: Thomas Bus Acceleration Tests

<table>
<thead>
<tr>
<th>Speed (mph)</th>
<th>Cumulative Acceleration Time (seconds)</th>
<th>Distance (feet)</th>
<th>Average Acceleration (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.7</td>
<td>4.2</td>
<td>59</td>
<td>0.182</td>
</tr>
<tr>
<td>29.9</td>
<td>11.7</td>
<td>328</td>
<td>0.117</td>
</tr>
<tr>
<td>37.4</td>
<td>18.6</td>
<td>656</td>
<td>0.092</td>
</tr>
<tr>
<td>42.2</td>
<td>23.2</td>
<td>984</td>
<td>0.083</td>
</tr>
<tr>
<td>46.0</td>
<td>29.1</td>
<td>1312</td>
<td>0.072</td>
</tr>
<tr>
<td>48.7</td>
<td>33.8</td>
<td>1640</td>
<td>0.066</td>
</tr>
</tbody>
</table>

*ARJ Volume 10 No. 5*

#### Table 2: Snowmobile Engine Deceleration Tests

<table>
<thead>
<tr>
<th>Speed (mph)</th>
<th>Average Engine Deceleration (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.16 – 0.18</td>
</tr>
<tr>
<td>40</td>
<td>0.24 – 0.25</td>
</tr>
</tbody>
</table>

*Snowmobile Accident Reconstruction; L & J*
Acceleration / Deceleration Rate

Acceleration (positive)/Deceleration (negative) is the rate of change of velocity with respect to time. Because velocity has the units of ft/sec, and time has the unit of seconds, the units for acceleration are:

Table 1: Acceleration / Deceleration Rates (ft/sec²)

<table>
<thead>
<tr>
<th>Speed Range</th>
<th>f</th>
<th>ft/sec²</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Passenger cars - normal acceleration</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Less than 20 mi/hr</td>
<td>0.15</td>
<td>4.8</td>
</tr>
<tr>
<td>20 to 40 mi/hr</td>
<td>0.10</td>
<td>3.2</td>
</tr>
<tr>
<td>Greater than 40 mi/hr</td>
<td>0.05</td>
<td>1.6</td>
</tr>
<tr>
<td><strong>Passenger cars – rapid acceleration</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Less than 20 mi/hr</td>
<td>0.30</td>
<td>9.7</td>
</tr>
<tr>
<td>20 to 40 mi/hr</td>
<td>0.15</td>
<td>4.8</td>
</tr>
<tr>
<td>Greater than 40 mi/hr</td>
<td>0.05</td>
<td>3.2</td>
</tr>
<tr>
<td><strong>Passenger cars – deceleration (coasting out of gear)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Less than 20 mi/hr</td>
<td>-0.01</td>
<td>0.3</td>
</tr>
<tr>
<td>20 to 40 mi/hr</td>
<td>-0.02</td>
<td>0.6</td>
</tr>
<tr>
<td>Greater than 40 mi/hr</td>
<td>-0.04</td>
<td>1.3</td>
</tr>
<tr>
<td><strong>Passenger cars – deceleration (engine braking high gear)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Less than 20 mi/hr</td>
<td>-0.04</td>
<td>1.3</td>
</tr>
<tr>
<td>20 to 40 mi/hr</td>
<td>-0.05</td>
<td>1.6</td>
</tr>
<tr>
<td>Greater than 40 mi/hr</td>
<td>-0.08</td>
<td>2.6</td>
</tr>
<tr>
<td><strong>Medium trucks – normal acceleration</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Less than 20 mi/hr</td>
<td>0.10</td>
<td>3.2</td>
</tr>
<tr>
<td>20 to 40 mi/hr</td>
<td>0.05</td>
<td>1.6</td>
</tr>
<tr>
<td>Greater than 40 mi/hr</td>
<td>0.03</td>
<td>1.0</td>
</tr>
<tr>
<td><strong>Large loaded trucks – normal acceleration</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Less than 20 mi/hr</td>
<td>0.05</td>
<td>1.6</td>
</tr>
<tr>
<td>20 to 40 mi/hr</td>
<td>0.03</td>
<td>1.0</td>
</tr>
<tr>
<td>Greater than 40 mi/hr</td>
<td>0.01</td>
<td>0.3</td>
</tr>
</tbody>
</table>
Table 2: Idle Acceleration From a Stop

<table>
<thead>
<tr>
<th>Speed Range</th>
<th>Forward</th>
<th>Reverse</th>
<th>Maximum Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average</td>
<td>Peak</td>
<td>Average</td>
</tr>
<tr>
<td>Mean</td>
<td>0.029</td>
<td>0.051</td>
<td>0.021</td>
</tr>
<tr>
<td>Median</td>
<td>0.03</td>
<td>0.05</td>
<td>0.02</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.009</td>
<td>0.019</td>
<td>0.006</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.01</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.04</td>
<td>0.09</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Table 3: Perception/Reaction Time

**Simple Reaction ~ Stop Light/Sign Acceleration**

0.20 – 0.30 seconds

*AIQ 23 p.16*
## Air Bag

### Deployment

<table>
<thead>
<tr>
<th>Speed</th>
<th>Deployment Time</th>
<th>Full Inflation Time</th>
<th>Inflated Deployment Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-5 mph</td>
<td>No Deployment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15-20 mph – Medium</td>
<td>40-60 Milliseconds</td>
<td>50-70 Milliseconds</td>
<td>≅ 200 mph</td>
</tr>
<tr>
<td>30-35 mph – Severe</td>
<td>10-20 Milliseconds</td>
<td>50-70 Milliseconds</td>
<td>≅ 200 mph</td>
</tr>
</tbody>
</table>

### Full Deployment

<table>
<thead>
<tr>
<th>Bag</th>
<th>Deployment Time</th>
<th>Deployment Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drivers Side</td>
<td>30 Milliseconds</td>
<td>≅ 200 mph</td>
</tr>
<tr>
<td>Passenger Side</td>
<td>60 Milliseconds</td>
<td>≅ 200 mph</td>
</tr>
</tbody>
</table>

### Commencement of Deflation

<table>
<thead>
<tr>
<th>Bag</th>
<th>Deployment Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drivers Side</td>
<td>75-150 Milliseconds</td>
</tr>
<tr>
<td>Passenger Side</td>
<td>75-150 Milliseconds</td>
</tr>
</tbody>
</table>

### Full Deflation

<table>
<thead>
<tr>
<th>Bag</th>
<th>Deployment Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drivers Side</td>
<td>1-2 Seconds</td>
</tr>
<tr>
<td>Passenger Side</td>
<td>1-2 Seconds</td>
</tr>
</tbody>
</table>

Occupant full engagement within 100 ms

Crash sensing in first 20 ms

Deployment 9-15 mph EBS for vehicle

Deployment within 15 degrees to either side of vehicle centerline

Side Airbags:

- Sensing in 1st 4-5 ms
- Full inflation within 20 ms
Drag Coefficients

A force that resists the relative motion or tendency to such motion of two bodies in contact. A number or symbol multiplying a variable or an unknown quantity in an algebraic term. A numerical measure of a physical or chemical property that is constant for a system under specified conditions such as the coefficient of friction.

Table 1: Friction Coefficients During Slide

* Level surface with all wheels locked and sliding.  

<table>
<thead>
<tr>
<th>Surface</th>
<th>Dry -30 MI/HR</th>
<th>Dry +30 MI/HR</th>
<th>Wet -30 MI/HR</th>
<th>Wet +30 MI/HR</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Portland Cement</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>New / Sharp</td>
<td>.80-.12</td>
<td>.70-.10</td>
<td>.50-.80</td>
<td>.40-.75</td>
</tr>
<tr>
<td>Traveled /</td>
<td>.60-.80</td>
<td>.60-.75</td>
<td>.45-.70</td>
<td>.45-.65</td>
</tr>
<tr>
<td>Worn</td>
<td>.55-.75</td>
<td>.50-.65</td>
<td>.45-.65</td>
<td>.45-.60</td>
</tr>
<tr>
<td>Traffic polished</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Asphalt / Tar</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>New / Sharp</td>
<td>.80-.12</td>
<td>.65-.10</td>
<td>.50-.80</td>
<td>.45-.75</td>
</tr>
<tr>
<td>Traveled /</td>
<td>.60-.80</td>
<td>.55-.80</td>
<td>.45-.70</td>
<td>.40-.65</td>
</tr>
<tr>
<td>Worn</td>
<td>.55-.75</td>
<td>.45-.75</td>
<td>.45-.65</td>
<td>.40-.60</td>
</tr>
<tr>
<td>Traffic polished</td>
<td>.50-.60</td>
<td>.40-.60</td>
<td>.30-.60</td>
<td>.25-.55</td>
</tr>
<tr>
<td>Excess tar</td>
<td>.63</td>
<td>.63</td>
<td>.30-.60</td>
<td>.25-.55</td>
</tr>
<tr>
<td>Chip/Seal</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Brick</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>New / Sharp</td>
<td>.75-.95</td>
<td>.60-.85</td>
<td>.50-.75</td>
<td>.45-.70</td>
</tr>
<tr>
<td>Traffic polished</td>
<td>.60-.80</td>
<td>.55-.75</td>
<td>.40-.70</td>
<td>.40-.60</td>
</tr>
<tr>
<td><strong>Stone Block</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>New / Sharp</td>
<td>.75-.10</td>
<td>.70-.90</td>
<td>.65-.90</td>
<td>.60-.85</td>
</tr>
<tr>
<td>Traffic polished</td>
<td>.50-.70</td>
<td>.45-.65</td>
<td>.30-.50</td>
<td>.25-.50</td>
</tr>
<tr>
<td><strong>Grass</strong></td>
<td>.30-.50</td>
<td>.30-.50</td>
<td>.20-.40</td>
<td>.20-.40</td>
</tr>
<tr>
<td><strong>Dirt</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loose packed</td>
<td>.40-.60</td>
<td>.40-.60</td>
<td>.30-.50</td>
<td>.30-.50</td>
</tr>
<tr>
<td>Mud</td>
<td></td>
<td></td>
<td>.40-.50</td>
<td>.40-.50</td>
</tr>
<tr>
<td><strong>Gravel</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Packed / Oiled</td>
<td>.55-.85</td>
<td>.55-.85</td>
<td>.40-.80</td>
<td>.40-.60</td>
</tr>
<tr>
<td>Loose</td>
<td>.40-.70</td>
<td>.40-.70</td>
<td>.45-.75</td>
<td>.45-.75</td>
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<tr>
<td>Truck escape ramps</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Cinders</strong></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Packed</td>
<td>.50-.70</td>
<td>.50-.70</td>
<td>.65-.75</td>
<td>.65-.75</td>
</tr>
<tr>
<td>Surface</td>
<td>Friction Coefficient</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>------------------</td>
<td>----------------------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Rolling Resistance ~ General</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Drive axle</td>
<td>.02-.03</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non drive axle</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normal inflation</td>
<td>.01</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Partial inflation</td>
<td>.013</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flat tire</td>
<td>.017</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Table 2: Rolling Resistance for Car Tires</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surface</td>
<td>Friction Coefficient</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>----------------------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concrete</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>New</td>
<td>.01</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Good</td>
<td>.015 - .017</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Worn</td>
<td>.02</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Asphalt</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>New</td>
<td>.0125</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Good</td>
<td>.0175</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Worn</td>
<td>.0225</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gravel</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Good (Macadam)</td>
<td>.015</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fair (Macadam)</td>
<td>.0225</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Poor (Macadam)</td>
<td>.0375</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pea Gravel (Truck Ramp)</td>
<td>.25 - .35</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Snow</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two inches</td>
<td>.025</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Four inches</td>
<td>.0375</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dirt</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Smooth</td>
<td>.045</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loose Sand</td>
<td>.05 - .30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mud</td>
<td>.0375 - .15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sage Brush, Firm Soil</td>
<td>.07 - .10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grass</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dry</td>
<td>.05 - .10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sand</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level, soft</td>
<td>.06 - .15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dune, loose</td>
<td>.16 - .30</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3; Rolling Resistance for Truck Tires
<table>
<thead>
<tr>
<th>Cobbles</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Ordinary</td>
<td></td>
<td>0.055</td>
</tr>
<tr>
<td>Poor</td>
<td></td>
<td>0.085</td>
</tr>
<tr>
<td>Snow</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 inches</td>
<td></td>
<td>0.025</td>
</tr>
<tr>
<td>4 inches</td>
<td></td>
<td>0.0375</td>
</tr>
<tr>
<td>Dirt</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Smooth</td>
<td></td>
<td>0.025</td>
</tr>
<tr>
<td>Loose Sand</td>
<td></td>
<td>0.0375</td>
</tr>
<tr>
<td>Mud</td>
<td></td>
<td>0.0375 - 0.15</td>
</tr>
<tr>
<td>Sand</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level, soft</td>
<td></td>
<td>0.06 - 0.15</td>
</tr>
<tr>
<td>Dune, loose</td>
<td></td>
<td>0.16 - 0.30</td>
</tr>
</tbody>
</table>

**Lane Change**

*Perception/Reaction Time Phase*

<table>
<thead>
<tr>
<th>Lane Change</th>
<th>5-7 Seconds</th>
</tr>
</thead>
</table>

*Bernard S. Abrams, O.D.*

**Truck Impact**

*Table 1: Acceleration Rates*

<table>
<thead>
<tr>
<th>Vehicle Type</th>
<th>ft/sec$^2$</th>
<th>(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Medium trucks – normal acceleration</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Less than 20 mi/hr</td>
<td>+3.2</td>
<td>+0.10</td>
</tr>
<tr>
<td>20 to 40 mi/hr</td>
<td>+1.6</td>
<td>+0.05</td>
</tr>
<tr>
<td>Greater than 40 mi/hr</td>
<td>+1.0</td>
<td>+0.03</td>
</tr>
<tr>
<td><strong>Loaded trucks - normal acceleration</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Less than 20 mi/hr</td>
<td>+3.2</td>
<td>+0.10</td>
</tr>
<tr>
<td>20 to 40 mi/hr</td>
<td>+1.6</td>
<td>+0.05</td>
</tr>
<tr>
<td>Greater than 40 mi/hr</td>
<td>+1.0</td>
<td>+0.03</td>
</tr>
</tbody>
</table>

*Table 2: Vehicle Braking Percentage with Proper Brake Adjustment*

<table>
<thead>
<tr>
<th>e Type</th>
<th>Percentage of Braking (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tractor/Semi Trailer (5 axle)</strong></td>
<td></td>
</tr>
<tr>
<td>% steer, 36% drives, 24% trailer</td>
<td></td>
</tr>
</tbody>
</table>
es (Cab over Engine tractor & twin 28's)

Trucks

Concrete Mixers (Caution: Limited Testing)

Homes (Caution: Limited Testing)

Commercial Buses

9 (Greyhound style)

Pass (including articulated city buses)

Vans

over Engine

Conventional

: Front axle brakes may slightly increase the driving coefficient; however the coefficient will still fall within a range of 0.3 - 0.4

UTION: If the vehicle is equipped with a brake proportioning valve, the percentage will increase dramatically to 80-85%

ails w/ BP-1 & BP-2 values

aters' w/BP-1

itelines (86+) w/WABCO 6 Channel anti-lock

(Westinghouse Air Brake Company)

Table 3: Center of Mass Height

<table>
<thead>
<tr>
<th>Bob Tail snub nose semi tractor</th>
<th>inches above ground</th>
</tr>
</thead>
<tbody>
<tr>
<td>Federal Regulations do not permit CM over 75 inches.</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Air Application Time

<table>
<thead>
<tr>
<th>Vehicle Type</th>
<th>Time (0-60 psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Range</td>
<td>0.25-0.50</td>
</tr>
<tr>
<td>Trucks, Buses, Truck Tractor</td>
<td>0.45</td>
</tr>
<tr>
<td>Dollies</td>
<td>0.35</td>
</tr>
<tr>
<td>Trailers</td>
<td>0.30</td>
</tr>
<tr>
<td>Light Laden Vehicles</td>
<td>0.25</td>
</tr>
</tbody>
</table>

* It is preferable for trailers to apply slightly ahead of tractors for stability.
Table 5: Air Release Time

<table>
<thead>
<tr>
<th>Vehicle Type</th>
<th>Time (95-5 psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trucks, Buses, Truck Tractor</td>
<td>0.55</td>
</tr>
<tr>
<td>Dollies</td>
<td>0.65</td>
</tr>
<tr>
<td>Trailers</td>
<td>0.65</td>
</tr>
</tbody>
</table>

* It is preferable for the tractor to release slightly ahead of the trailer.

Light Luminous

Luminous Levels

<table>
<thead>
<tr>
<th>Condition</th>
<th>Foot Lamberts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bright Sky ~ Clear</td>
<td>7,000</td>
</tr>
<tr>
<td>Overcast Skies</td>
<td>400</td>
</tr>
<tr>
<td>Optimum Luminance</td>
<td>400</td>
</tr>
<tr>
<td>Pavement Glare</td>
<td>25,000</td>
</tr>
</tbody>
</table>

Fixed Lighting Levels

<table>
<thead>
<tr>
<th>Type</th>
<th>Lumens/watt</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Pressure Sodium (Streetlight)</td>
<td>60-130</td>
</tr>
<tr>
<td>Low Pressure Sodium</td>
<td>78-150</td>
</tr>
<tr>
<td>Mercury Lamp (Streetlight)</td>
<td>37-54</td>
</tr>
<tr>
<td>Metal Halide</td>
<td>65-110</td>
</tr>
<tr>
<td>Incandescent Lamps (Vehicles)</td>
<td>11-18</td>
</tr>
<tr>
<td>Halogen Lamps (Vehicles)</td>
<td>20-22</td>
</tr>
<tr>
<td>Fluorescent</td>
<td>58-69</td>
</tr>
<tr>
<td>Locomotive Headlight</td>
<td>260,000 - 290,000 candela</td>
</tr>
</tbody>
</table>

Lighting Distances

<table>
<thead>
<tr>
<th>Type</th>
<th>Low Beam (ft)</th>
<th>High Beam (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passenger Car</td>
<td>120 (range)</td>
<td>150-175</td>
</tr>
</tbody>
</table>
Motorcycle Impact

Articulated Single Track Vehicles

Table 1: Motorcycle Friction Coefficient Values

<table>
<thead>
<tr>
<th>Motorcycles</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Free rolling</td>
<td>.01-.02</td>
</tr>
<tr>
<td>Front/Rear Full Lockup</td>
<td>8-1.1</td>
</tr>
<tr>
<td>Moderate/heavy front brake application with rear wheel lockup</td>
<td>$f_c = (\mu / 2 + \mu) / 2$</td>
</tr>
<tr>
<td>Front Wheel Only</td>
<td></td>
</tr>
<tr>
<td>Clean, dry surface</td>
<td>.65-.70</td>
</tr>
<tr>
<td>Rear Wheel Only</td>
<td></td>
</tr>
<tr>
<td>Clean, dry surface</td>
<td>.35-.45</td>
</tr>
<tr>
<td>Soft soil, sand</td>
<td>.90-1.2</td>
</tr>
<tr>
<td>Hard soil</td>
<td>.70</td>
</tr>
<tr>
<td>Motorcycle on Side</td>
<td></td>
</tr>
<tr>
<td>Asphalt, soft</td>
<td>1.0 +</td>
</tr>
<tr>
<td>Asphalt, hard</td>
<td>.30-.48</td>
</tr>
<tr>
<td>Portland cement</td>
<td>.40-.75</td>
</tr>
<tr>
<td>Concrete</td>
<td>.45-.53</td>
</tr>
<tr>
<td>Gravel</td>
<td>.70-1.0</td>
</tr>
<tr>
<td>Sand</td>
<td>1.5-1.6</td>
</tr>
<tr>
<td>Hard soil</td>
<td>.70</td>
</tr>
<tr>
<td>Light scratching</td>
<td>.30-.40</td>
</tr>
<tr>
<td>Crash bar contact only</td>
<td>.20-.40</td>
</tr>
<tr>
<td>Heavy scratching/gouge ¼ inch depth</td>
<td>.50-.60</td>
</tr>
<tr>
<td>Lubricated surface from motorcycle fluids</td>
<td>.20</td>
</tr>
</tbody>
</table>
**Rider**

| Rolling – tumbling Sliding on Asphalt, Portland Cement, Concrete | .85-1.0 |
| Riding Leather Polyester Cotton/wool (street clothing) Grass | .60-.70 .70 .70-.90 .79 |

**Table 2: Rider Launch angles**

| **Standard motorcycle** | 5 – 10° |
| Operator | 10 – 25° |
| Passenger | |

| **Café Style motorcycle** | 15 - 30° |
| Operator | 45° |
| Passenger | |

**Table 3: Idle RPM**

| **Standard motorcycle** | 1000 – 1500 rpm |
| **Café Style motorcycle** | 2000 rpm |

**Railroad Crossing Impacts**

**Table 1: Approximate Stopping Distance**

| **150-Car Freight Train Approximate Stopping Distance** | 30 mph = 3,500 feet or 2/3 of a mile 50 mph = 8,000 feet or 1½ miles |
| **8-Car Passenger Train Approximate Stopping Distance** | 60 mph = 3,500 feet or 2/3 of a mile 79 mph = 6,000 feet or 1 1/8 miles |

**Table 2: Fixed Lighting Levels**

| **Lighting Source** | **Candle Power** | **Distance** |
| Locomotive Headlight | 260,000 – 290,000 candela | 800 feet |
### Table 3: Operating Speeds of Track Class

<table>
<thead>
<tr>
<th>Track Class</th>
<th>Freight Train ~ mph</th>
<th>Passenger Train ~ mph</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 1 Track</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>Class 2 Track</td>
<td>25</td>
<td>30</td>
</tr>
<tr>
<td>Class 3 Track</td>
<td>40</td>
<td>60</td>
</tr>
<tr>
<td>Class 4 Track</td>
<td>60</td>
<td>80</td>
</tr>
<tr>
<td>Class 5 Track</td>
<td>80</td>
<td>90</td>
</tr>
<tr>
<td>Class 6 Track</td>
<td>110</td>
<td>110</td>
</tr>
</tbody>
</table>

### Table 4: Audible Levels

<table>
<thead>
<tr>
<th>Source</th>
<th>dBA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distinguishable</td>
<td>9 – 10 dBA above threshold</td>
</tr>
<tr>
<td>Insertion loss for vehicle</td>
<td>30 dBA</td>
</tr>
<tr>
<td>Inside Vehicle: 50 mph ~ Average</td>
<td></td>
</tr>
<tr>
<td>Windows closed / no radio</td>
<td>72 dBA</td>
</tr>
<tr>
<td>Horn from Locomotive: 100 ft</td>
<td>96 dBA 49CFR229</td>
</tr>
<tr>
<td>Interior cab of Truck ~ Average</td>
<td>85+ dBA</td>
</tr>
</tbody>
</table>

### Table 5: Example Findings

<table>
<thead>
<tr>
<th>A Time Before Impact</th>
<th>B Outside dBA Level</th>
<th>C Insertion dBA Loss</th>
<th>D Signal dBA Inside</th>
<th>E Operating dBA Level</th>
<th>F Signal-to-Noise Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 Seconds</td>
<td>92</td>
<td>30</td>
<td>62</td>
<td>73</td>
<td>-11</td>
</tr>
<tr>
<td>4 Seconds</td>
<td>102</td>
<td>30</td>
<td>72</td>
<td>73</td>
<td>-1</td>
</tr>
<tr>
<td>3 Seconds</td>
<td>100</td>
<td>30</td>
<td>70</td>
<td>73</td>
<td>-3</td>
</tr>
<tr>
<td>2.5 Seconds</td>
<td>103</td>
<td>30</td>
<td>73</td>
<td>73</td>
<td>0</td>
</tr>
<tr>
<td>2 Second</td>
<td>104.5</td>
<td>30</td>
<td>74.5</td>
<td>73</td>
<td>1.5</td>
</tr>
<tr>
<td>1.5 Seconds</td>
<td>107</td>
<td>30</td>
<td>77</td>
<td>73</td>
<td>4</td>
</tr>
<tr>
<td>1 Second</td>
<td>111.5</td>
<td>30</td>
<td>81.5</td>
<td>73</td>
<td>8.5</td>
</tr>
</tbody>
</table>

*Train Accident Reconstruction p.228 ~ Loumiet*
Table 6: Friction Coefficients

<table>
<thead>
<tr>
<th>Brake Shoes</th>
<th>Wheel to Pad Friction Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cobra Shoes</td>
<td>0.3 df</td>
</tr>
</tbody>
</table>

Table 7: Brake Cylinder Pressure

<table>
<thead>
<tr>
<th>Train</th>
<th>Pressure psig</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freight</td>
<td>60 -70</td>
</tr>
</tbody>
</table>

**Rollover**

*Any vehicle rotation of 90 degrees or more about any true longitudinal or lateral axis.*

Table 1: Rollover Thresholds

<table>
<thead>
<tr>
<th>Vehicle Type</th>
<th>CG Height In</th>
<th>Track Width In</th>
<th>Rollover Threshold g</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cars</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sports</td>
<td>18-20</td>
<td>50-60</td>
<td>1.2-1.7</td>
</tr>
<tr>
<td>Compact</td>
<td>20-23</td>
<td>50-60</td>
<td>1.1-1.5</td>
</tr>
<tr>
<td>Luxury</td>
<td>20-24</td>
<td>60-65</td>
<td>1.2-1.6</td>
</tr>
<tr>
<td><strong>Trucks</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pickup</td>
<td>30-35</td>
<td>65-70</td>
<td>0.9-1.1</td>
</tr>
<tr>
<td>Medium</td>
<td>45-55</td>
<td>65-75</td>
<td>0.6-0.8</td>
</tr>
<tr>
<td>Heavy</td>
<td>60-85</td>
<td>70-72</td>
<td>0.4-0.6</td>
</tr>
<tr>
<td><strong>Passenger van</strong></td>
<td>30-40</td>
<td>65-70</td>
<td>0.8-1.1</td>
</tr>
<tr>
<td><strong>Tractor-Trailer ~ 5 Axle</strong></td>
<td>Low CG</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>High CG</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td><strong>Semi tanker ~ 5 Axle</strong></td>
<td></td>
<td>0.35</td>
<td></td>
</tr>
<tr>
<td><strong>Tanker ~ Liquefied Gas</strong></td>
<td></td>
<td>0.26</td>
<td></td>
</tr>
<tr>
<td><strong>Logging Truck</strong></td>
<td></td>
<td></td>
<td>0.23-0.31</td>
</tr>
</tbody>
</table>

*Rollover of Heavy Commercial Vehicles ~ SAE*
Table 2: Suspension Roll Center Heights

UMTRI measurements
RPM Speed

Table 1: Vehicle Transmission Gear Ratio

<table>
<thead>
<tr>
<th>Vehicle Type</th>
<th>Overall Gear Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passenger cars</td>
<td>3 - 8 : 1</td>
</tr>
<tr>
<td>Small trucks</td>
<td>3 - 8 : 1</td>
</tr>
<tr>
<td>Large trucks</td>
<td>4 – 12 : 1</td>
</tr>
<tr>
<td>Buses</td>
<td>4 – 12 : 1</td>
</tr>
<tr>
<td>Tractors</td>
<td>5 – 18 : 1</td>
</tr>
<tr>
<td>Semi trailers</td>
<td>5 – 18 : 1</td>
</tr>
</tbody>
</table>

Table 2: Tire-Surface Friction Values

<table>
<thead>
<tr>
<th>Engine Braking</th>
<th>(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low gear</td>
<td>.20-.40</td>
</tr>
<tr>
<td>High gear</td>
<td>.20</td>
</tr>
</tbody>
</table>

Temperatures

Freezing Points

<table>
<thead>
<tr>
<th>Product</th>
<th>Temperature</th>
<th>Alias</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>32° F</td>
<td></td>
</tr>
<tr>
<td>Sodium Chloride &amp; Water</td>
<td>-6° F</td>
<td>Salt</td>
</tr>
<tr>
<td>80 Proof Liquor</td>
<td>-8° F</td>
<td></td>
</tr>
<tr>
<td>Methyl Alcohol</td>
<td>-20° F</td>
<td>Windshield washer solution</td>
</tr>
<tr>
<td>Propylene Glycol</td>
<td>-25° F</td>
<td></td>
</tr>
<tr>
<td>50/50 Antifreeze</td>
<td>-34° F</td>
<td></td>
</tr>
<tr>
<td>Magnesium Chloride</td>
<td>-35° F</td>
<td>Type of road salt</td>
</tr>
<tr>
<td>Calcium Chloride</td>
<td>-67° F</td>
<td></td>
</tr>
<tr>
<td>Potassium Acetate</td>
<td>-76° F</td>
<td>Type of road salt</td>
</tr>
<tr>
<td>70/30 Antifreeze</td>
<td>-84° F</td>
<td></td>
</tr>
<tr>
<td>Pure Methyl Alcohol</td>
<td>-193° F</td>
<td>Pure methyl or ethyl alcohol</td>
</tr>
</tbody>
</table>
### Boiling Points

<table>
<thead>
<tr>
<th>Product</th>
<th>Temperature</th>
<th>Water Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brake Fluid DOT 3</td>
<td>401°F</td>
<td>281</td>
</tr>
<tr>
<td>DOT 4</td>
<td>446°F</td>
<td>326</td>
</tr>
<tr>
<td>DOT 5</td>
<td>500°F</td>
<td>380</td>
</tr>
</tbody>
</table>

### Time

A non-spatial continuum in which events occur in apparently irreversible succession from the past through the present to the future. An interval separating two points on this continuum; a duration.

**Table 1: Perception/Reaction Time**

<table>
<thead>
<tr>
<th>Simple Reaction</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stop Light/Sign ~ Acceleration</td>
<td>0.20 – 0.30 seconds</td>
</tr>
<tr>
<td>Red Brake/Stop Light</td>
<td>0.55 seconds</td>
</tr>
</tbody>
</table>

**Table 2: Collision Time**

<table>
<thead>
<tr>
<th>Collision Type</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frontal</td>
<td>80 – 120 Milliseconds</td>
</tr>
<tr>
<td>Pole &amp; Angle</td>
<td>96 – 180 Milliseconds (20-50 % longer)</td>
</tr>
</tbody>
</table>

**Table 3: Movement**

<table>
<thead>
<tr>
<th>Movement</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exit Vehicle</td>
<td>3.5 seconds</td>
</tr>
</tbody>
</table>

The dynamics of bodies moving relative to gases, especially the interaction of moving objects with the atmosphere.
### Table 1: Standard Atmospheric conditions for Different Altitudes

<table>
<thead>
<tr>
<th>Altitude (ft)</th>
<th>Pressure (inches Hg)</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Standard</td>
</tr>
<tr>
<td>Sea Level</td>
<td>29.92</td>
<td>59.0</td>
</tr>
<tr>
<td>1000</td>
<td>28.45</td>
<td>55.4</td>
</tr>
<tr>
<td>2000</td>
<td>27.41</td>
<td>51.9</td>
</tr>
<tr>
<td>3000</td>
<td>26.41</td>
<td>48.3</td>
</tr>
<tr>
<td>4000</td>
<td>25.45</td>
<td>44.7</td>
</tr>
<tr>
<td>5000</td>
<td>24.52</td>
<td>41.2</td>
</tr>
<tr>
<td>6000</td>
<td>23.62</td>
<td>37.6</td>
</tr>
<tr>
<td>7000</td>
<td>22.75</td>
<td>34.1</td>
</tr>
<tr>
<td>8000</td>
<td>21.91</td>
<td>30.5</td>
</tr>
</tbody>
</table>

### Table 2a: Aerodynamic Drag Coefficients

<table>
<thead>
<tr>
<th>Vehicle Type</th>
<th>Drag Coefficient ($C_d$)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Passenger Cars</strong></td>
<td></td>
</tr>
<tr>
<td>1960-1970's</td>
<td>.40-.50</td>
</tr>
<tr>
<td>1980-1990's</td>
<td>.30-.40</td>
</tr>
<tr>
<td><strong>Trucks</strong></td>
<td></td>
</tr>
<tr>
<td>1980's Pickup</td>
<td>.45</td>
</tr>
<tr>
<td>Straight</td>
<td>.65-.75</td>
</tr>
<tr>
<td>Semitrailer, Tank</td>
<td>.65-.75</td>
</tr>
<tr>
<td>Tank w/pup, Flatbed w/load</td>
<td>.75-.85</td>
</tr>
<tr>
<td>Doubles</td>
<td>.75-.85</td>
</tr>
<tr>
<td>Car Haulers</td>
<td>1.0</td>
</tr>
<tr>
<td>Van</td>
<td>.44</td>
</tr>
<tr>
<td><strong>Busses</strong></td>
<td>.50-.80</td>
</tr>
<tr>
<td><strong>Motorcycles</strong></td>
<td>.50-.80</td>
</tr>
<tr>
<td>w/Fairing</td>
<td>.55</td>
</tr>
<tr>
<td><strong>Bicycles</strong></td>
<td>.90</td>
</tr>
</tbody>
</table>
Table 2b: Aerodynamic Drag Coefficients

<table>
<thead>
<tr>
<th>Vehicle Type</th>
<th>Drag Coefficient ( (C_D) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
</tr>
<tr>
<td>Experimental</td>
<td>.17</td>
</tr>
<tr>
<td>Sports</td>
<td>.27</td>
</tr>
<tr>
<td>Performance</td>
<td>.32</td>
</tr>
<tr>
<td>60's Muscle</td>
<td>.38</td>
</tr>
<tr>
<td>Sedan</td>
<td>.34</td>
</tr>
<tr>
<td>Motorcycle</td>
<td>.50</td>
</tr>
<tr>
<td>Truck</td>
<td>.60</td>
</tr>
<tr>
<td>Tractor-Trailer</td>
<td>.60</td>
</tr>
</tbody>
</table>

Table 2: Speed Rating

<table>
<thead>
<tr>
<th>Rate Character</th>
<th>Speed mph</th>
<th>Rate Character</th>
<th>Speed mph</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>31</td>
<td>P</td>
<td>93</td>
</tr>
<tr>
<td>C</td>
<td>37</td>
<td>Q</td>
<td>99</td>
</tr>
<tr>
<td>D</td>
<td>40</td>
<td>R</td>
<td>106</td>
</tr>
<tr>
<td>E</td>
<td>43</td>
<td>S</td>
<td>112</td>
</tr>
<tr>
<td>F</td>
<td>50</td>
<td>T</td>
<td>118</td>
</tr>
<tr>
<td>G</td>
<td>56</td>
<td>U</td>
<td>124</td>
</tr>
<tr>
<td>J</td>
<td>62</td>
<td>H</td>
<td>130</td>
</tr>
<tr>
<td>K</td>
<td>68</td>
<td>V</td>
<td>149</td>
</tr>
<tr>
<td>L</td>
<td>75</td>
<td>Y</td>
<td>188</td>
</tr>
<tr>
<td>M</td>
<td>81</td>
<td>ZR</td>
<td>OVER 149 MPH</td>
</tr>
<tr>
<td>N</td>
<td>87</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table 3: Load Rating

<table>
<thead>
<tr>
<th>Rating</th>
<th>Poundage</th>
<th>Rating</th>
<th>Poundage</th>
<th>Rating</th>
<th>Poundage</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>852 lbs</td>
<td>83</td>
<td>1074 lbs</td>
<td>91</td>
<td>1356 lbs</td>
</tr>
<tr>
<td>76</td>
<td>882 lbs</td>
<td>84</td>
<td>1102 lbs</td>
<td>92</td>
<td>1389 lbs</td>
</tr>
<tr>
<td>77</td>
<td>908 lbs</td>
<td>85</td>
<td>1135 lbs</td>
<td>93</td>
<td>1433 lbs</td>
</tr>
<tr>
<td>78</td>
<td>937 lbs</td>
<td>86</td>
<td>1168 lbs</td>
<td>94</td>
<td>1477 lbs</td>
</tr>
<tr>
<td>79</td>
<td>963 lbs</td>
<td>87</td>
<td>1201 lbs</td>
<td>95</td>
<td>1521 lbs</td>
</tr>
<tr>
<td>80</td>
<td>992 lbs</td>
<td>88</td>
<td>1235 lbs</td>
<td>96</td>
<td>1565 lbs</td>
</tr>
<tr>
<td>81</td>
<td>1019 lbs</td>
<td>89</td>
<td>1279 lbs</td>
<td>97</td>
<td>1609 lbs</td>
</tr>
<tr>
<td>82</td>
<td>1047 lbs</td>
<td>90</td>
<td>1323 lbs</td>
<td>98</td>
<td>1653 lbs</td>
</tr>
</tbody>
</table>

### Table 1; Load & Inflation Table
Tire Load Limits at Various Cold Inflation Points

<table>
<thead>
<tr>
<th>RIM DIA. SERIES</th>
<th>TIRE SIZE DESIGNATION</th>
<th>INFLATIONS (psi)</th>
<th>LOAD INDEX</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>20 23 26 29 32 35</td>
<td>STANDARD LOAD (lbs)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>LOAD INDEX</td>
</tr>
<tr>
<td>70</td>
<td>P265/70R15</td>
<td>177 190 201 212 222 233</td>
<td>110</td>
</tr>
<tr>
<td></td>
<td>P275/70R15</td>
<td>189 202 214 227 238 246</td>
<td>112</td>
</tr>
<tr>
<td>65</td>
<td>P185/65R15</td>
<td>904 959 101 106 112 116</td>
<td>86</td>
</tr>
<tr>
<td></td>
<td>P195/65R15</td>
<td>981 104 111 116 123 127</td>
<td>89</td>
</tr>
<tr>
<td></td>
<td>P205/65R15</td>
<td>106 114 121 127 133 140</td>
<td>92</td>
</tr>
<tr>
<td></td>
<td>P215/65R15</td>
<td>115 123 131 137 145 151</td>
<td>95</td>
</tr>
<tr>
<td></td>
<td>P235/65R15</td>
<td>134 144 152 160 168 176</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>P255/65R15</td>
<td>155 165 175 185 194 202</td>
<td>105</td>
</tr>
<tr>
<td></td>
<td>P295/65R15</td>
<td>199 213 227 239 250 260</td>
<td>114</td>
</tr>
<tr>
<td>15</td>
<td>P185/60R15</td>
<td>838 893 948 100 104 110</td>
<td>84</td>
</tr>
<tr>
<td></td>
<td>P195/60R15</td>
<td>915 981 103 109 114 119</td>
<td>87</td>
</tr>
<tr>
<td></td>
<td>P205/60R15</td>
<td>992 105 112 119 124 130</td>
<td>90</td>
</tr>
<tr>
<td></td>
<td>P215/60R15</td>
<td>108 114 122 129 134 141</td>
<td>93</td>
</tr>
<tr>
<td></td>
<td>P225/60R15</td>
<td>115 124 131 138 145 152</td>
<td>95</td>
</tr>
</tbody>
</table>

245
<table>
<thead>
<tr>
<th>RIM DIA.</th>
<th>TIRE SIZE DESIGNATION</th>
<th>INFLATIONS (psi)</th>
<th>LOAD INDEX</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>80D16</td>
<td>860</td>
<td>915</td>
</tr>
<tr>
<td>75</td>
<td>75S16</td>
<td>149</td>
<td>159</td>
</tr>
<tr>
<td></td>
<td>75S16</td>
<td>160</td>
<td>173</td>
</tr>
<tr>
<td></td>
<td>P245/75 16</td>
<td>173</td>
<td>185</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td></td>
<td>P265/75 16</td>
<td>198</td>
<td>212</td>
</tr>
<tr>
<td>70</td>
<td>P215/70 16</td>
<td>130</td>
<td>138</td>
</tr>
<tr>
<td></td>
<td>P225/70 16</td>
<td>140</td>
<td>149</td>
</tr>
<tr>
<td></td>
<td>P235/70 16</td>
<td>151</td>
<td>160</td>
</tr>
<tr>
<td></td>
<td>P245/70 16</td>
<td>162</td>
<td>173</td>
</tr>
<tr>
<td></td>
<td>P255/70 16</td>
<td>173</td>
<td>185</td>
</tr>
<tr>
<td></td>
<td>P265/70 16</td>
<td>185</td>
<td>198</td>
</tr>
<tr>
<td></td>
<td>P275/70 16</td>
<td>197</td>
<td>211</td>
</tr>
<tr>
<td>65</td>
<td>P215/65 16</td>
<td>121</td>
<td>129</td>
</tr>
<tr>
<td></td>
<td>P225/65 16</td>
<td>130</td>
<td>140</td>
</tr>
<tr>
<td></td>
<td>P255/65 16</td>
<td>162</td>
<td>173</td>
</tr>
<tr>
<td></td>
<td>P265/65 16</td>
<td>173</td>
<td>184</td>
</tr>
<tr>
<td></td>
<td>P275/65 16</td>
<td>184</td>
<td>197</td>
</tr>
<tr>
<td></td>
<td>P285/65 16</td>
<td>196</td>
<td>209</td>
</tr>
<tr>
<td></td>
<td>P295/65 16</td>
<td>208</td>
<td>222</td>
</tr>
<tr>
<td>60</td>
<td>P205/60 16</td>
<td>103</td>
<td>111</td>
</tr>
<tr>
<td></td>
<td>P215/60 16</td>
<td>112</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td>P225/60 16</td>
<td>121</td>
<td>130</td>
</tr>
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<td></td>
<td>P235/60 16</td>
<td>131</td>
<td>140</td>
</tr>
<tr>
<td></td>
<td>P285/60 16</td>
<td>181</td>
<td>194</td>
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<tr>
<td>55</td>
<td>P205/55 16</td>
<td>970</td>
<td>103</td>
</tr>
<tr>
<td></td>
<td>P215/55 16</td>
<td>104</td>
<td>111</td>
</tr>
<tr>
<td></td>
<td>P225/55 16</td>
<td>112</td>
<td>120</td>
</tr>
<tr>
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